TU Clausthal
Clausthal University of Technology

Abstraction for Model Checking Modular Interpreted Systems over ATL

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Outline

Motivation

Local Communities

Nowadays **Social Networks**:

- exchange of information,
- groups of interests, and
- explicit use of a computer or smart phone.

Local Communities: **social networks** + **real social networks**

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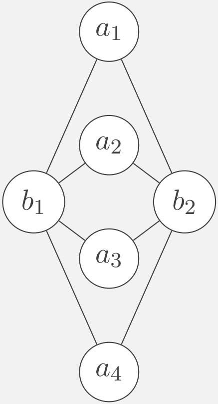
Communication between Agents

Example 1 (Communicating Agents)



- Agents of team B are not allowed to send a message back.
- If an agent b_j has received a message from a_i then it has to send it to some agent a_k in the following round, $k > i$.

Is it possible for team A to ensure that a_4 will know the message eventually?

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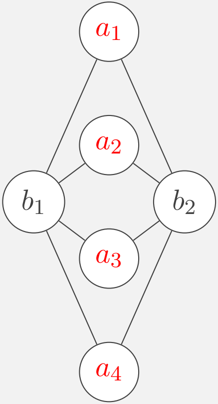

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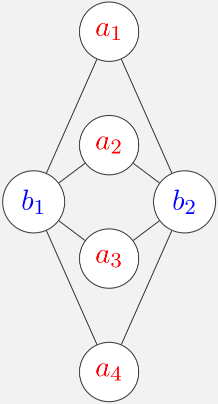

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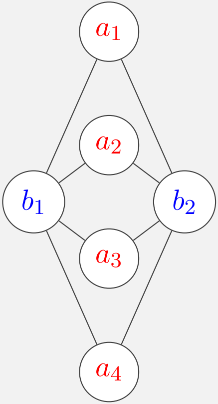

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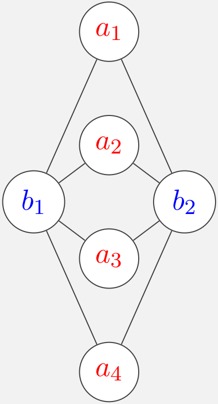

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- deal with big systems?
- formulate minimal properties (fairness, liveness, safety)?
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  Model Checking

Modular Interpreted System

Modular Interpreted System

Idea:

- Several **loosely** connected components.
- Work is done in the components.
- Little **interaction** between the components.

Modular Interpreted System

Definition 2 (Modular Interpreted System (MIS))

A MIS is a tuple $S = (\mathbb{A}\mathrm{gt}, Act, \mathcal{I}n)$ where:

- $\mathbb{A}\mathrm{gt} = \{a_1, \ldots, a_k\}$ agents,
- Act actions,
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Example 3

- $\mathbb{A}\mathrm{gt} = \{a_1, a_2, a_3, a_4, b_1, b_2\}$,
- $Act = \{\mathrm{send_x} \mid \mathrm{x} \in \mathbb{A}\mathrm{gt}\} \cup \{\mathrm{noop}\}$
- $\mathcal{I}n = \{\mathbf{nothing}, \mathbf{m_{a_1}}, \mathbf{m_{a_2}}, \mathbf{m_{a_3}}, \mathbf{m_{a_4}}\} \cup$ $\mathrm{P}(\{\mathbf{m_{a_i b_j}} \mid i \in \{1, \ldots, 4\}, j \in \{1, 2\}\})$

MIS Agent I

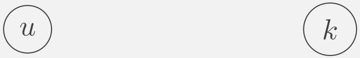
Definition 4 (MIS Agent)

Each agent has the following **internal structure**:

$$a_i = (St_i, \quad , \qquad , \quad , \quad , \qquad)$$

The global state space is defined as $St := St_1 \times \cdots \times St_k$.

Example 5 (Agent a_1)



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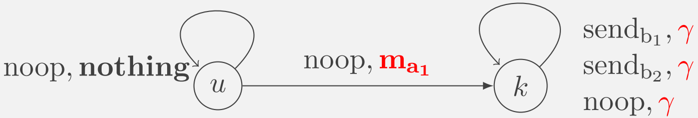

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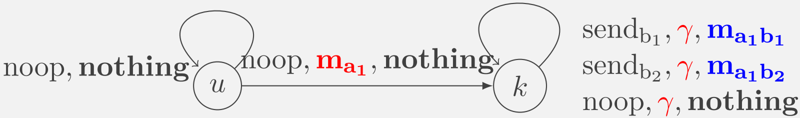

MIS Agent II

Example 6 (Agents b_j)

- 256 states
- Every state is labeled with $\mathsf{known}_{\mathsf{b_j}}$ if the agent received some time ago the message from a_i.
- Others are labeled with $\mathsf{unknown}_{\mathsf{b_j}}$.
- While the agent is waiting for a message it does nothing.
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- **Modular** (removing, replacing of agents)
- **Interaction** reduced to an abstract **interaction symbol**
- **Computational ground**

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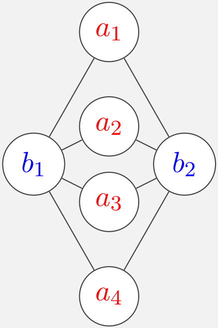

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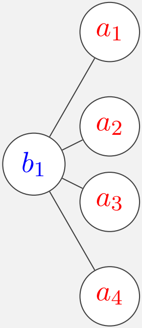

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Specification Logic: ATL

Alternating-time Temporal Logic

Definition 8 (Alternating-time temporal Logic (ATL))

The language of plain ATL is defined over the non-empty sets:

- Π of **Propositions** $\mathsf{p} \in \Pi$
- $\mathbb{A}\mathrm{gt}$ of **Agents** $A \subseteq \mathbb{A}\mathrm{gt}$

$$\varphi ::= \mathsf{p} \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\langle A \rangle\rangle \mathbf{X}\varphi \mid \langle\langle A \rangle\rangle \mathbf{G}\varphi \mid \langle\langle A \rangle\rangle \varphi \mathbf{U} \varphi.$$

Main Idea: cooperation modalities

- $\langle\langle A \rangle\rangle \mathbf{X}\varphi$ "coalition A has a collective strategy to enforce in the next step φ"

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Example 9

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$$S, \mathrm{q} \models \langle\langle A\rangle\rangle(\top \mathbf{U} \mathsf{known}_{\mathsf{a}_4})$$

with $A = \{a_1, a_2, a_3, a_4\}$ and q is the global state where a_1 is in state k, all other agents from team A are in state u and the agents from team B are in state $(\emptyset, \emptyset)$

Abstraction for MIS

Basic Idea I

Partitioning the local state space by using handcrafted equivalence relation:

- Reducing all equivalent states to just one
- Agents get fewer choices and the opponents more choices
- Influence symbols of the abstract states:
  - all influence symbols that are an outcome of executing a action in each concrete state
- Local transition function:

  Input: abstract state, action, influence symbol
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Abstraction Relation

Definition 10 (Abstraction Relation)

An **abstraction relation** for a MIS is a product $\equiv = \equiv_1 \times \cdots \times \equiv_k$ where each $\equiv_i \subseteq St_i \times St_i$ is an equivalence relation for the states St_i of agent a_i.

For $q \in St_i$, we write $[q]_{\equiv_i}$ for the equivalence class of the local state q with respect to $\equiv_i$. And for $q \in St = St_1 \times \cdots \times St_k$, we write $[q]_{\equiv}$ for the equivalence class of the global state q.

Definition 11 (Abstraction for MIS)

For a MIS $S = (\mathbb{A}\mathrm{gt}, Act, \mathcal{I}n)$, an abstraction relation $\equiv$ for S and a set of **favored** agents $A \subseteq \mathbb{A}\mathrm{gt}$ we define the **abstraction** of S with respect to $\equiv$ and A as the MIS

$$S^A_\equiv := (\mathbb{A}\mathrm{gt}', Act, \mathcal{I}n)$$

where $\mathbb{A}\mathrm{gt}' := \{a'_1, \ldots, a'_k\}$ and $a'_i = (St'_i, d'_i, out'_i, in'_i, o'_i, \Pi'_i, \pi'_i)$ with

- $St'_i \;:=\; \{[q]_{\equiv_i} \mid q \in St_i\}$
- $d'_i([q]_{\equiv_i}) \;:=\; \begin{cases} \bigcap_{q' \in [q]_{\equiv_i}} d_i(q') & \text{for } a_i \in A \\ \bigcup_{q' \in [q]_{\equiv_i}} d_i(q') & \text{for } a_i \notin A \end{cases}$
- $out'_i([q]_{\equiv_i}, \alpha) \;:=\; \bigcup\limits_{q' \in [q]_{\equiv_i}} out_i(q', \alpha)$

for all $q \in St_i$, $\alpha \in Act$, $\gamma, \gamma_1, \ldots, \gamma_k \in \mathcal{I}n$ and $p_i \in \Pi'_i$.

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- $in'_i([q]_{\equiv_i}, \gamma_1, \ldots, \gamma_{k-1}) := \bigcup_{q' \in [q]_{\equiv_i}} in_i(q', \gamma_1, \ldots, \gamma_{k-1})$
- $o'_i([q]_{\equiv_i}, \alpha, \gamma) := \bigcup_{q' \in [q]_{\equiv_i}} \{[q'']_{\equiv_i} \mid q'' \in o_i(q', \alpha, \gamma)\}$
- $\Pi'_i := \Pi_i \cap \{p_i \mid \forall q \in \pi_i(p_i) : \forall q' \in [q]_{\equiv_i} : q' \in \pi_i(p_i)\}$
- $\pi'_i(p_i) := \{[q]_{\equiv_i} \mid q \in \pi_i(p_i)\}$

for all $q \in St_i$, $\alpha \in Act$, $\gamma, \gamma_1, \ldots, \gamma_k \in \mathcal{I}n$ and $p_i \in \Pi'_i$.

Example for Abstraction

Example 13 (Agents b_j)

- 256 states

$$
\begin{aligned}
S_i &:= \{(R,N) \mid \emptyset \neq R \subseteq \{r_1,\dots,r_i\}, n_i \in N\} \setminus \bigcup_{j=1}^{i-1} S_j \\
&\ \text{for } i = 1,\dots,3 \\
S_{\text{rest}} &:= \{(R,N) \mid (R,N) \notin S_1 \cup \dots \cup S_3\}
\end{aligned}
$$

Result: **4 states**

The Model Checking Algorithm

Idea

- **Input**:
  - MIS S
  - $init$ of global states of S
  - ATL formula φ
  - for each quantifier subformulae an abstraction relation
- **Output**:
  - true : iff $S, q \models \varphi$ for all $q \in init$
  - unknown : we do not know whether S satisfies φ or not

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Algorithm: modelcheck

Algorithm *modelcheck*$(S, init, \varphi, (\equiv_\psi)_{\psi \in \mathrm{qsf}(\varphi)})$:
Let $\varphi = \lambda(\theta_1, \ldots, \theta_n, \ell_1, \ldots, \ell_m)$ where

- λ Boolean formula (conjunctions and disjunctions only),
- $\theta_1, \ldots, \theta_n$ are arbitrary ATL formulae each beginning with a quantifier, i.e. each θ_i is of the form $\langle\langle B \rangle\rangle \theta_i'$, and
- $\ell_1, \ldots, \ell_m$ are literals.
- For all $i \in \{1, \ldots, n\}$ do:
  - Set $W_i := label(\theta_i, \equiv_{\theta_i})$.
  - Set $S := S[w_i, W_i]$, a new global proposition w_i is introduced in S and it is labeled exactly in the states in W_i.
- If $S, s \models \lambda(w_1, \ldots, w_n, \ell_1, \ldots, \ell_m)$ for all $s \in init$ then return true. Otherwise return unknown.

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Let $\varphi = \lambda(\theta_1, \ldots, \theta_n, \ell_1, \ldots, \ell_m)$ where

- λ Boolean formula (conjunctions and disjunctions only),
- $\theta_1, \ldots, \theta_n$ are arbitrary ATL formulae each beginning with a quantifier, i.e. each θ_i is of the form $\langle\langle B \rangle\rangle \theta_i'$, and
- $\ell_1, \ldots, \ell_m$ are literals.
- For all $i \in \{1, \ldots, n\}$ do:
  Set $W_i :=$ *label*$(\theta_i, \equiv_\psi)$.
  Set $S := S(w_i, W_i)$, a new global proposition w_i is introduced in S and it is labeled exactly in the states in W_i.
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Algorithm *label*$(\psi, \equiv)$:
Let $\psi = \neg^{\psi}\langle\langle A\rangle\rangle \mathbf{Y} \lambda(\theta_1, \ldots, \theta_n, \ell_1, \ldots, \ell_m)$ where

- $\neg^{\psi}$ is $\neg$ if ψ begins with a negation and it is the empty string elsewise,
- $\mathbf{Y} \in \{\mathbf{X}, \mathbf{G}, \mathbf{U}\}$,
- λ is a monotone Boolean formula,
- $\theta_1, \ldots, \theta_n$ are arbitrary ATL formulae each beginning with a quantifier or a negation directly followed by a quantifier, and
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Algorithm: label II

- Construct the abstraction

$$S' := \begin{cases} S_{\equiv}^{[\![\psi]\!]} & \text{if } \psi \text{ does not begin with a negation} \\ S_{\equiv}^{\mathbb{A}\mathrm{gt}\setminus[\![\psi]\!]} & \text{if } \psi \text{ does begin with a negation} \end{cases}$$

- For all $i \in \{1, \dots, n\}$ do:
  - Set $W_i := \{[q]_{\equiv} \mid \forall q' \in [q]_{\equiv} : q' \in \mathit{label}(\theta_i, \equiv_{\theta_i})\}$.
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- Compute the set W' of global states of S' (note that these are global states of the system abstracted with $\equiv$) satisfying ψ, i.e. $W' :=$

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- Return $W := \{q \in St \mid [q]_{\equiv} \in W'\}$.

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Complexity and Soundness

Theorem 14

Algorithm modelcheck$(S, init, \varphi, (\equiv_\psi)_{\psi \in \mathrm{qsf}(\varphi)})$ *runs in time*

$$O\left(|init| + |S| \cdot |\varphi|\right) \cdot 2^{O\left(\sum_{\psi \in \mathrm{qsf}(\varphi)} \left|S^{[\![\psi]\!]}_{\equiv_\psi}\right|\right)}$$

where $|S|$ *denotes the size of the* MIS S *in a compact representation. The cardinality of the global state space of* S *may then be upto* $2^{\Theta(|S|)}$.

Theorem 15

Algorithm modelcheck is sound, i.e. if modelcheck$(S, init, \varphi,$ $(\equiv_\psi)_{\psi \in \mathrm{qsf}(\varphi)})$ *outputs* true *then* $S, q \models \varphi$ *for all* $q \in init$.

Conclusion

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- **Modular Interpreted Systems** facilitates **modularity**
- **ATL** allows to talk about strategic properties
- The **abstraction for MIS** :
  - is sound
  - allows to deal with bigger Systems
    (depending on the equivalent relations)

Thank you for your attention!

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