A logical method for temporal knowledge representation and reasoning

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February 12, 2003

Outline

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Problem statement

- **Objective:** Modelling domains that **change in time**

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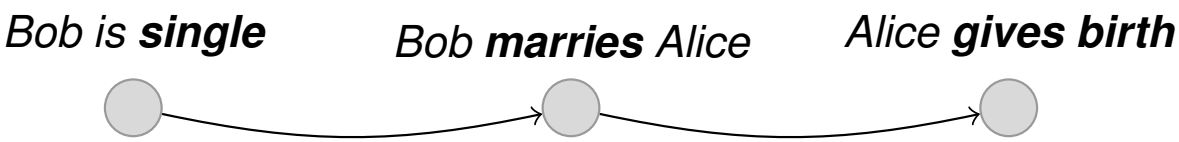

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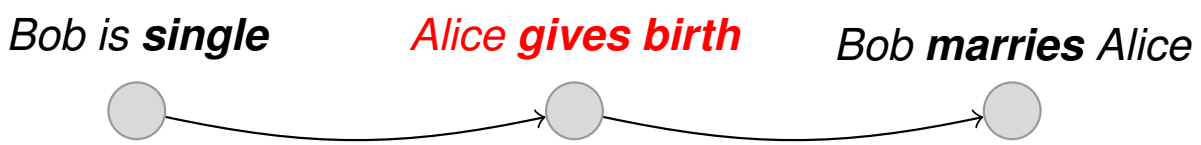

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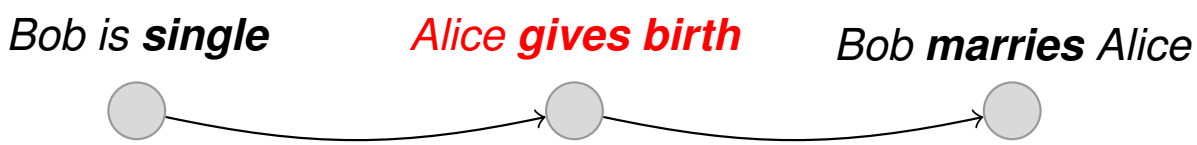

Distinctive features of our domains:

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- The domain's evolution is **non-Markovian**.

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How do we **represent**

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How do we **represent** and **reason** about changing domains in an **efficient** manner ?

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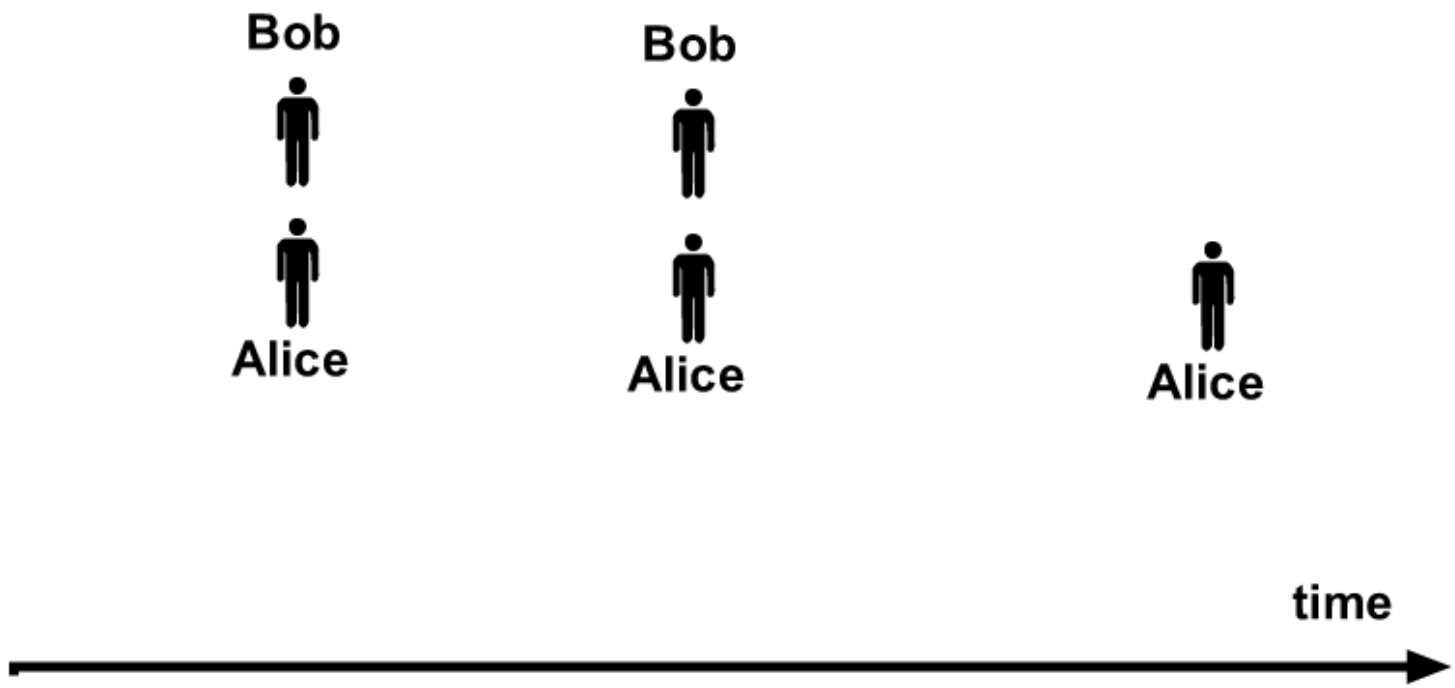

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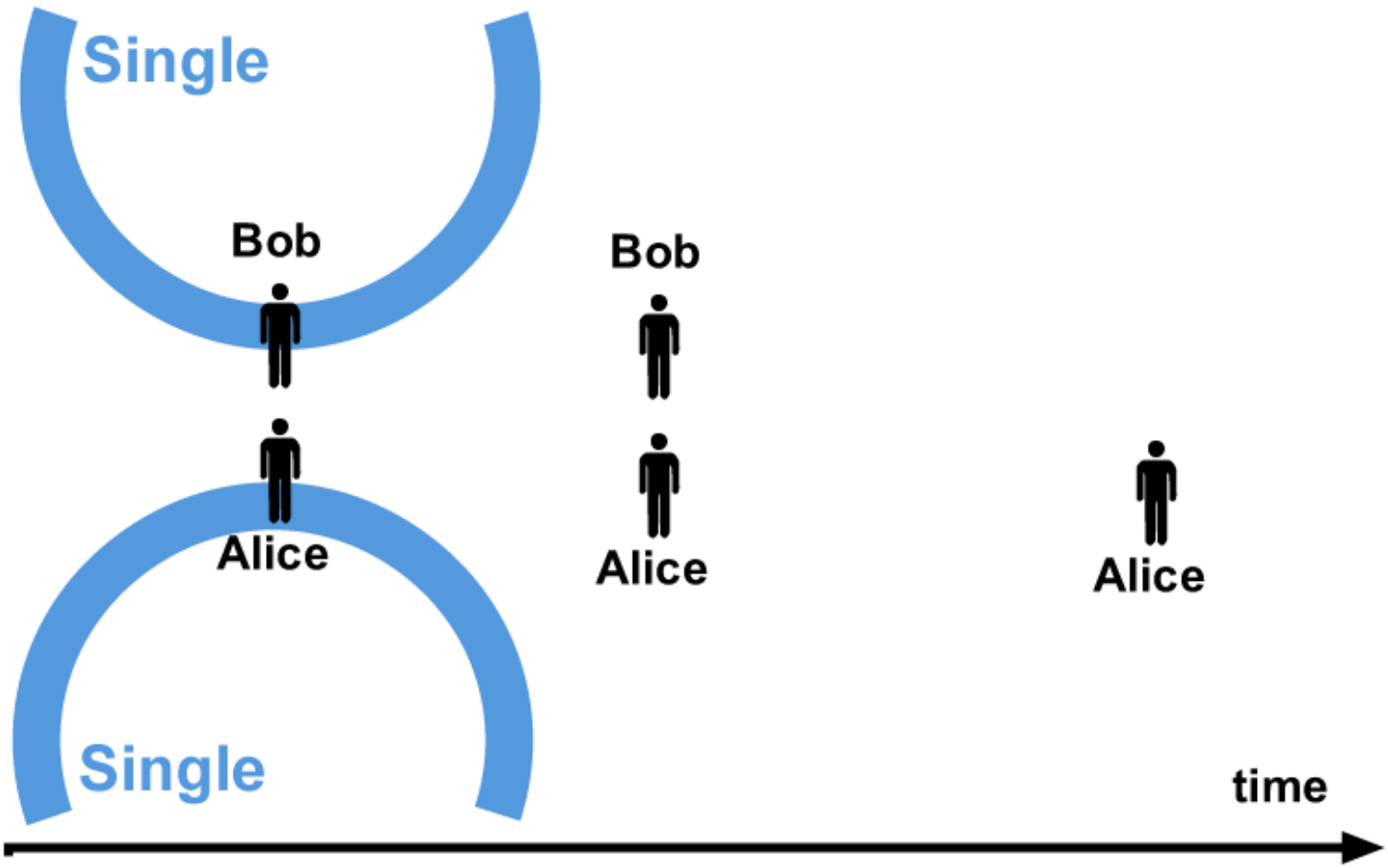

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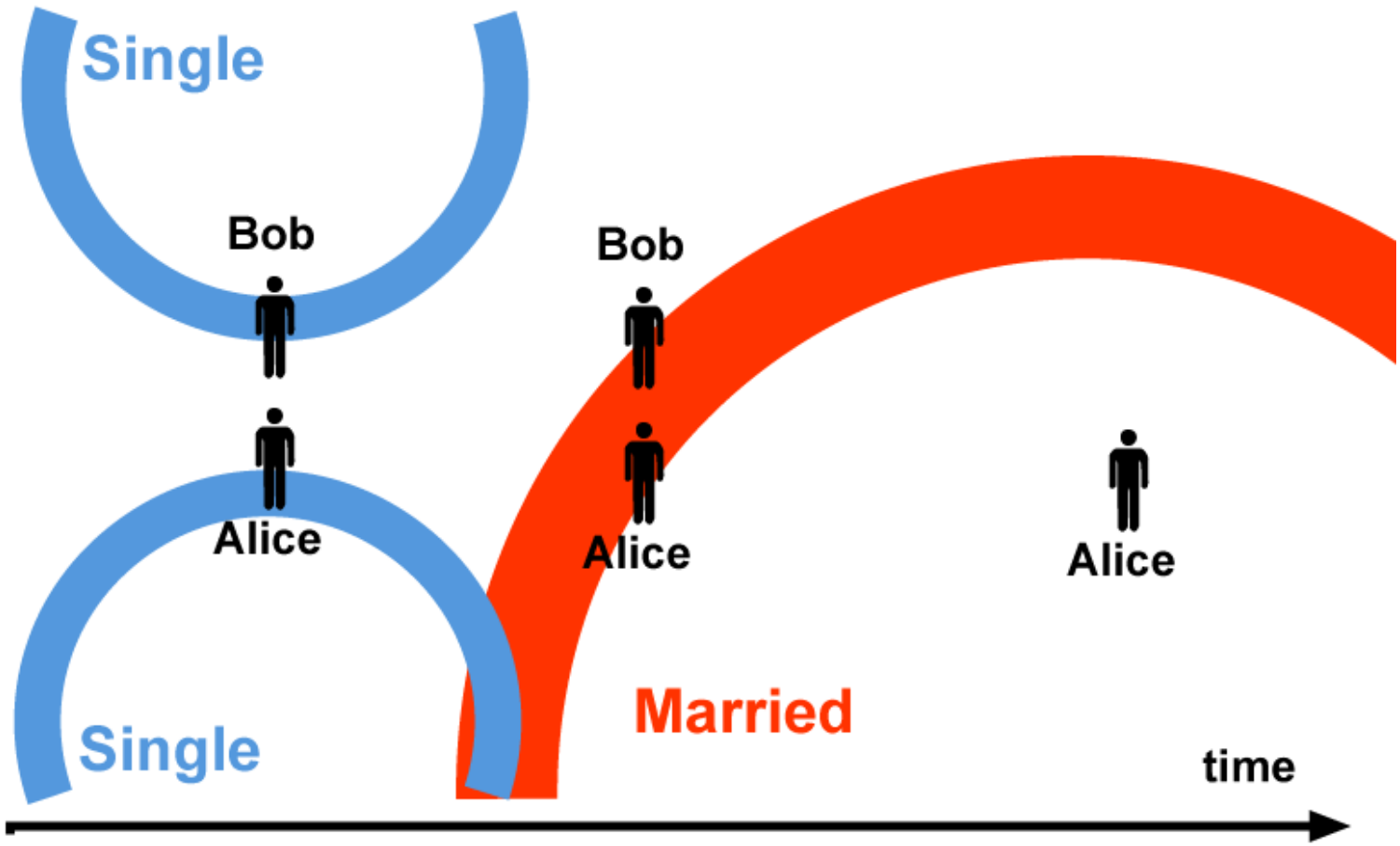

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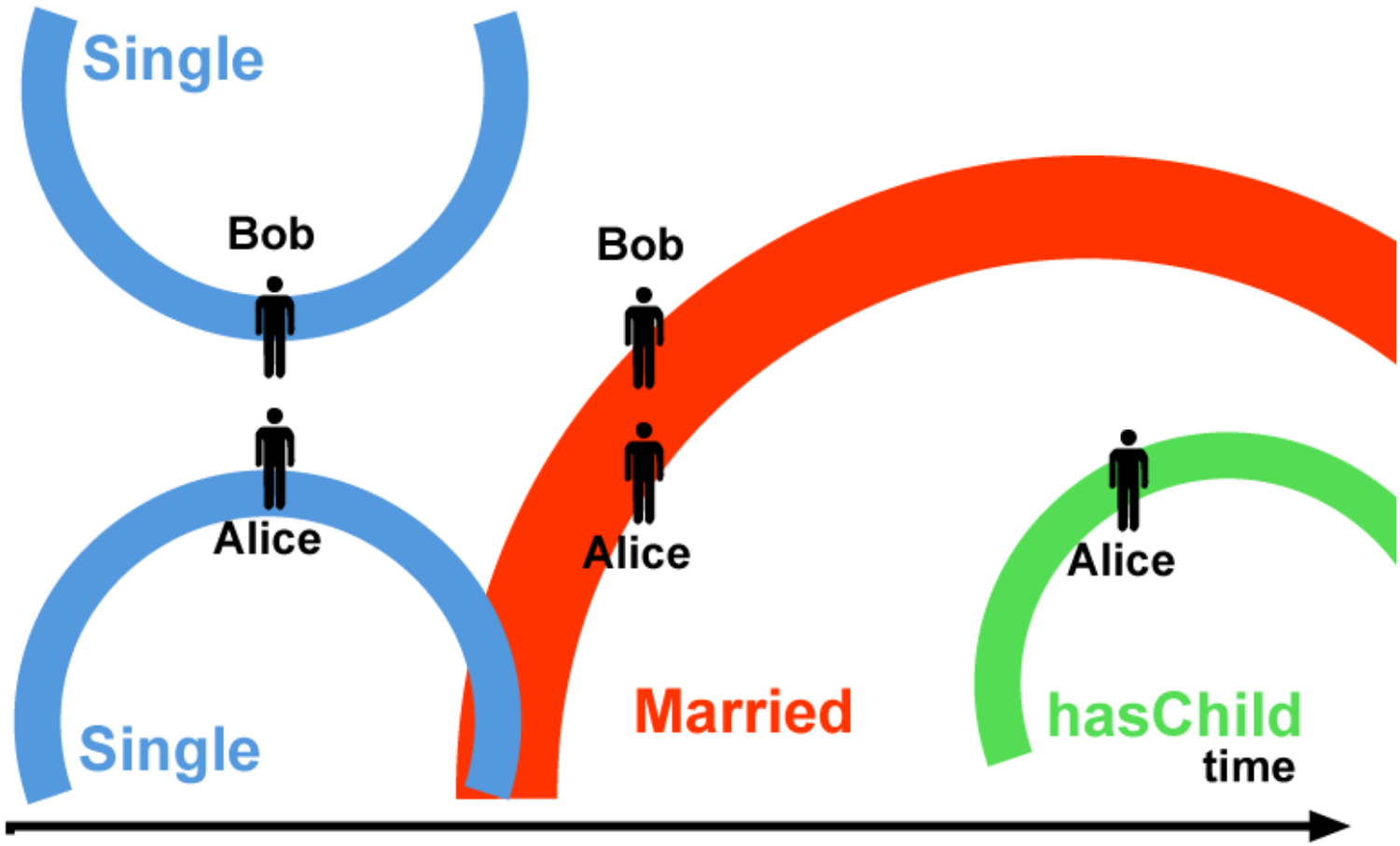
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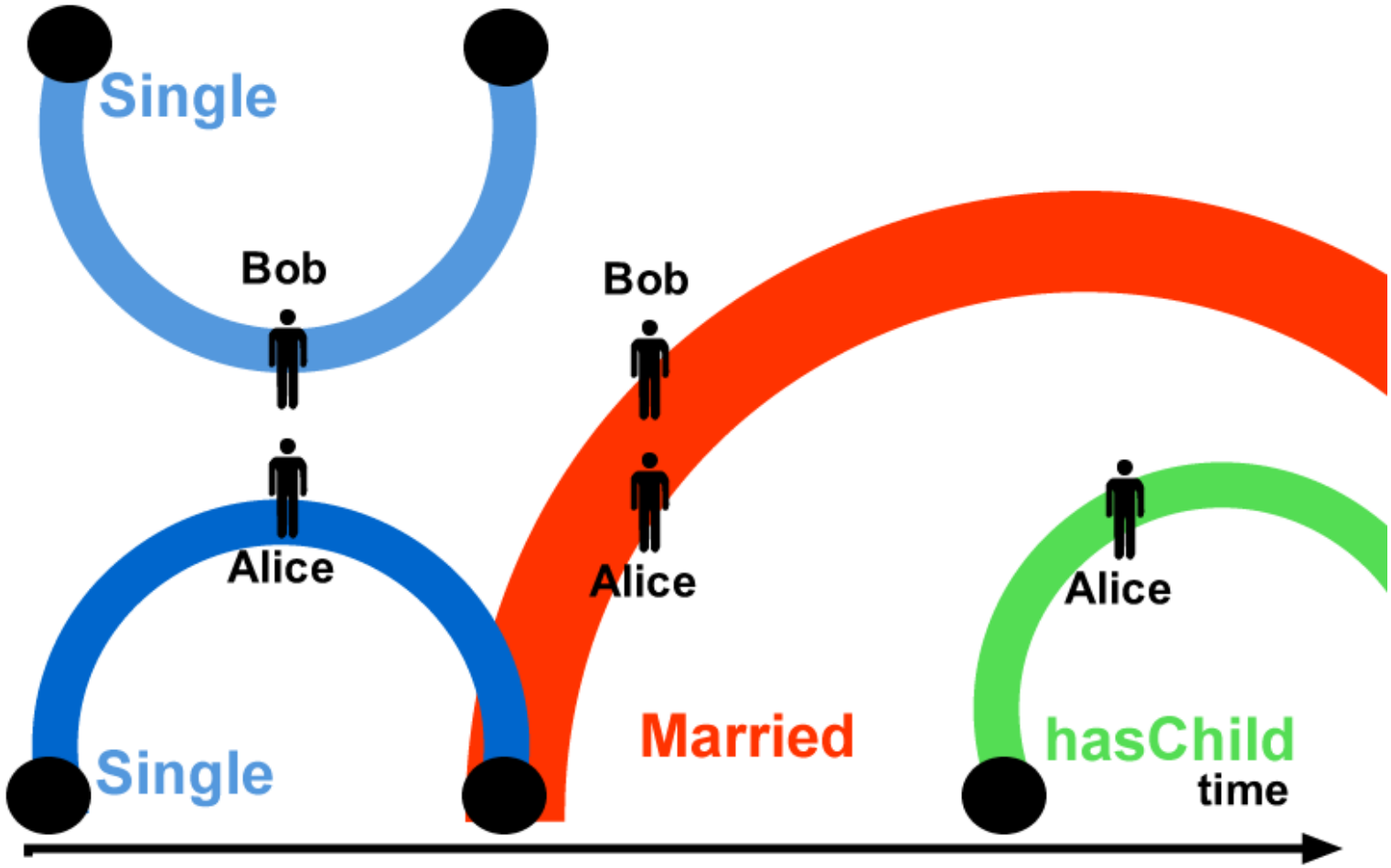
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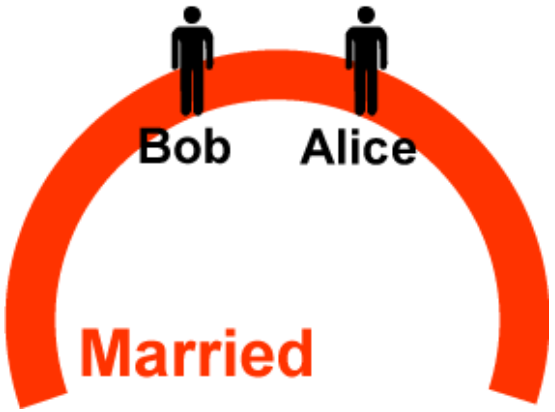

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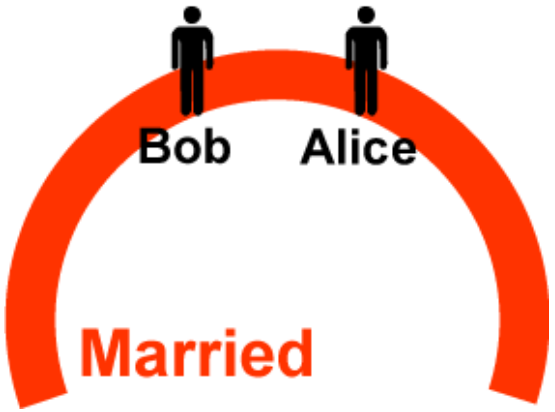

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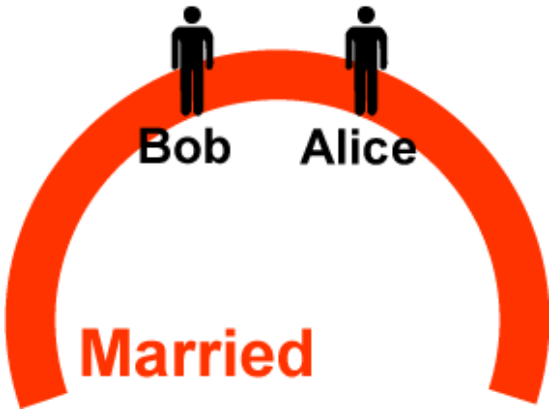

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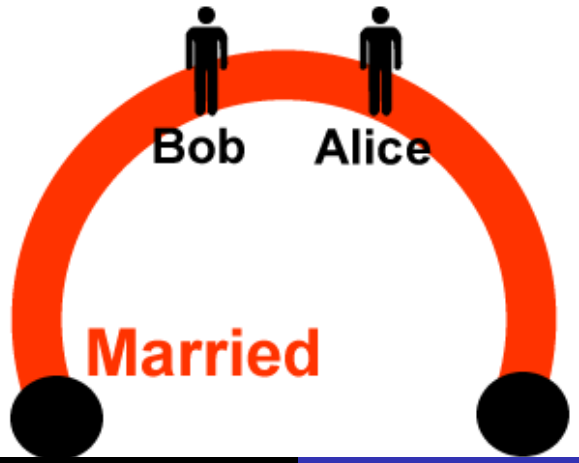

Temporal graphs

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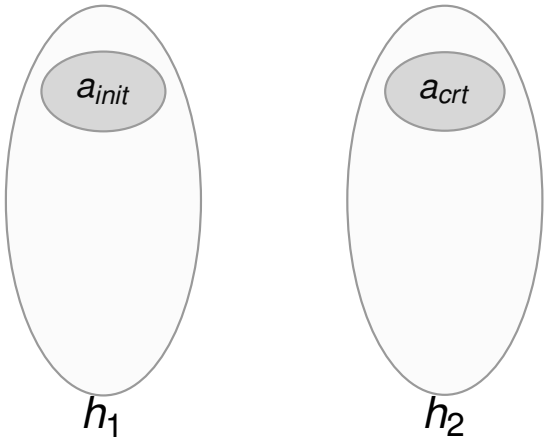

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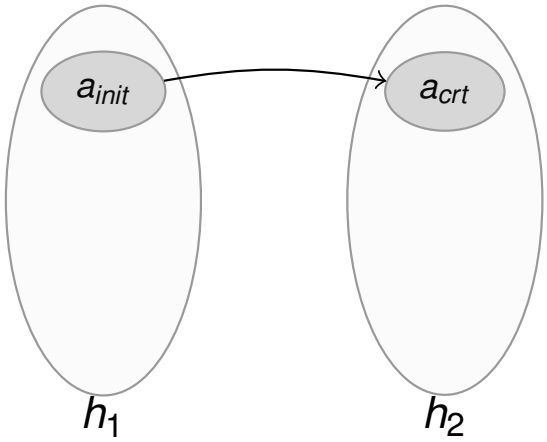

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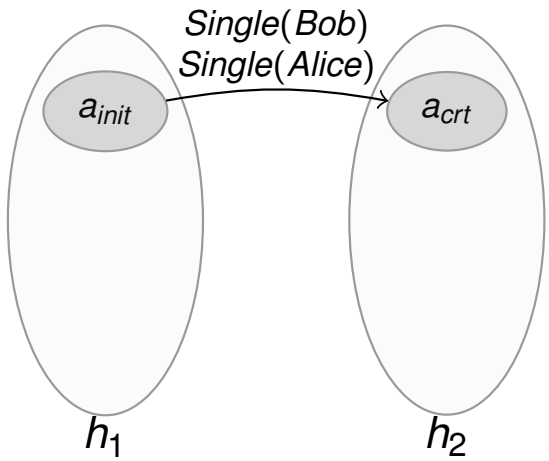

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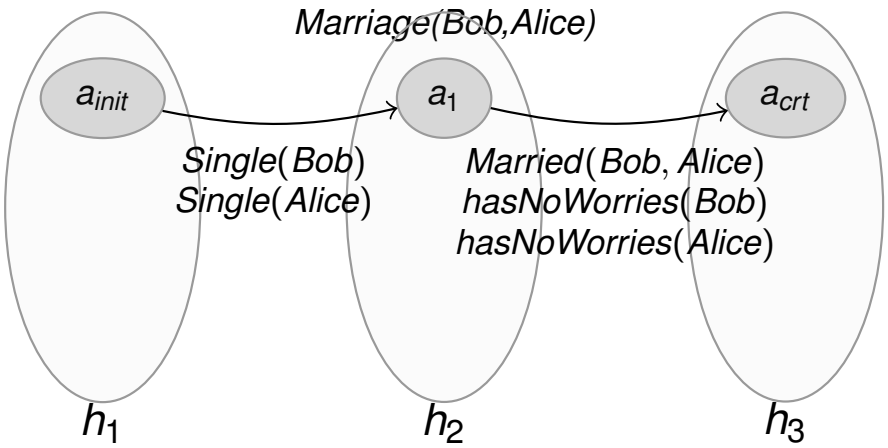

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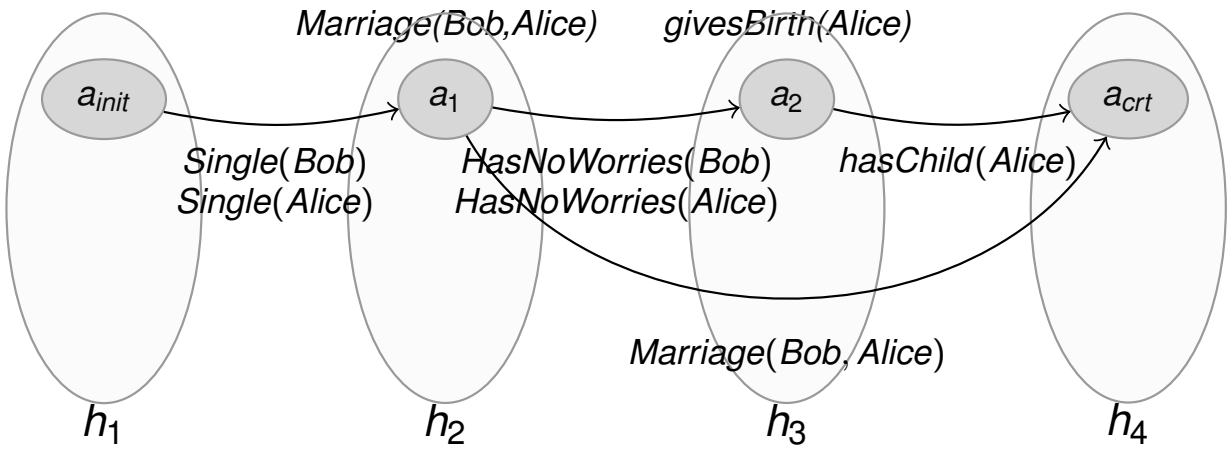

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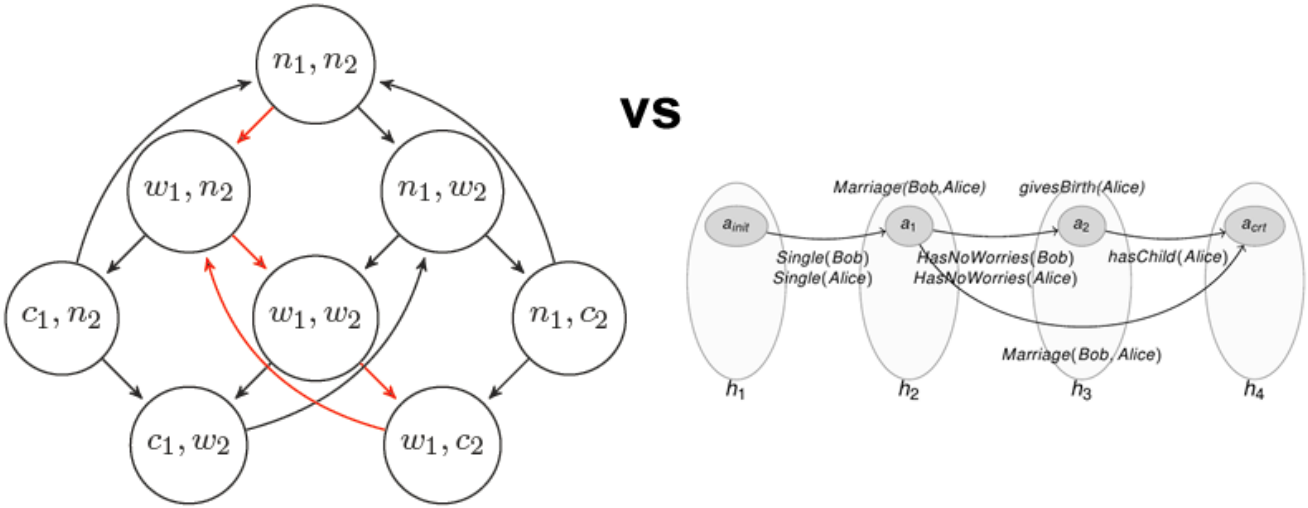
Temporal graphs and Kripke Structures



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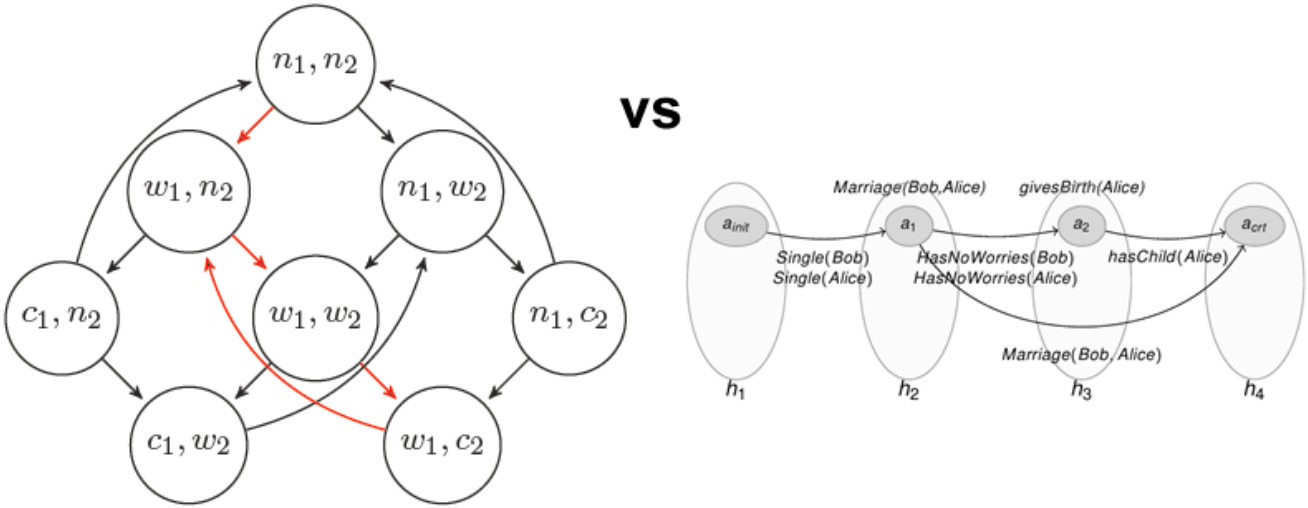

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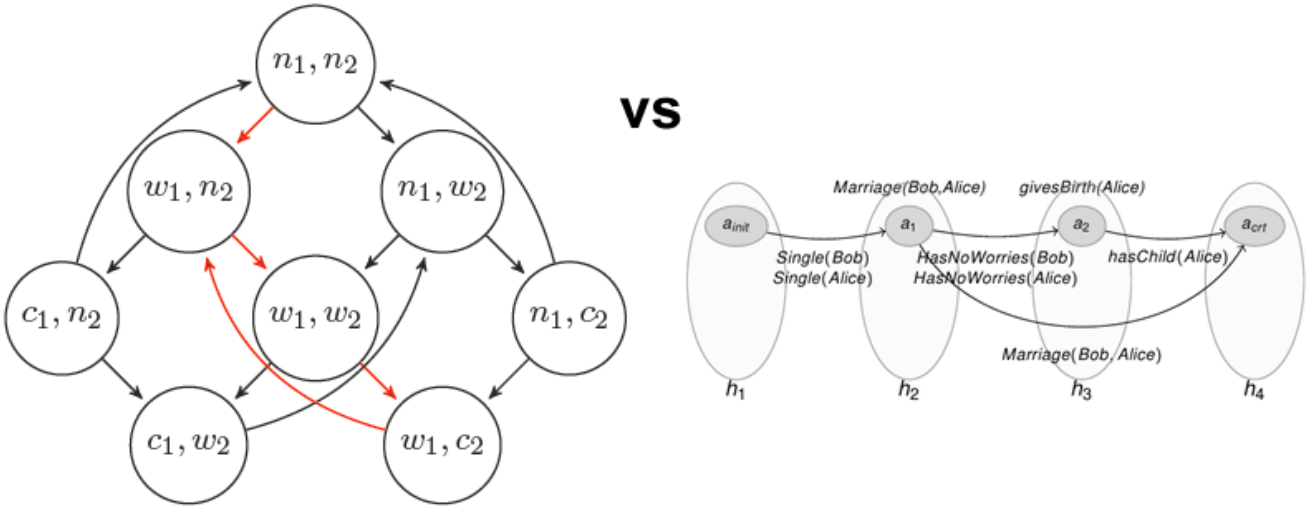

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- temporal graphs are **behavioural structures** (similar to **paths** from a KS

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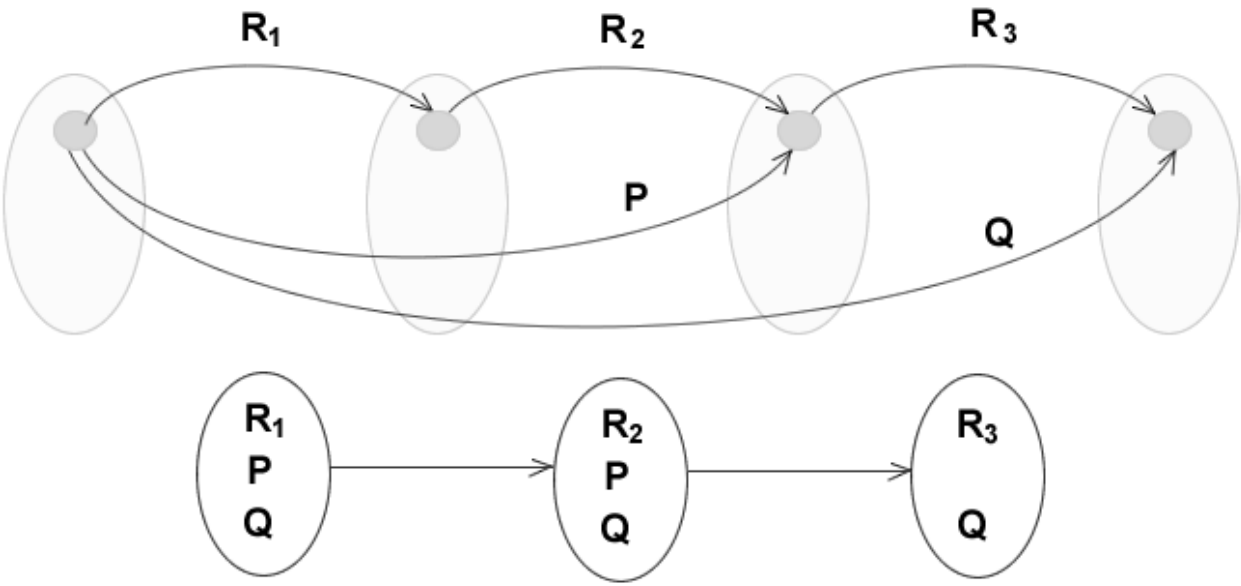

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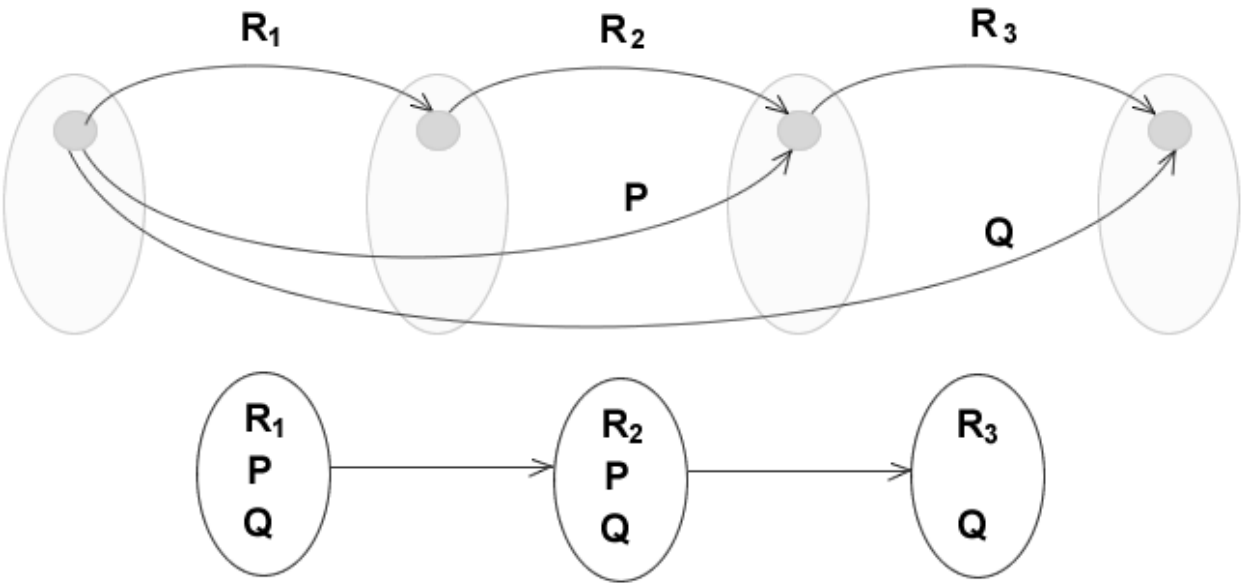

Temporal graphs **versus** Kripke Structures:

- **explosion** of the number of labellings (scalability problem)

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There exists no formula from First Order Logic (FOL) which can **express**: **connectivity** or the **existence of a path** between two components of a temporal graph

The Ehrenfeucht-Fraïssé method (connectivity)

fix a natural number k

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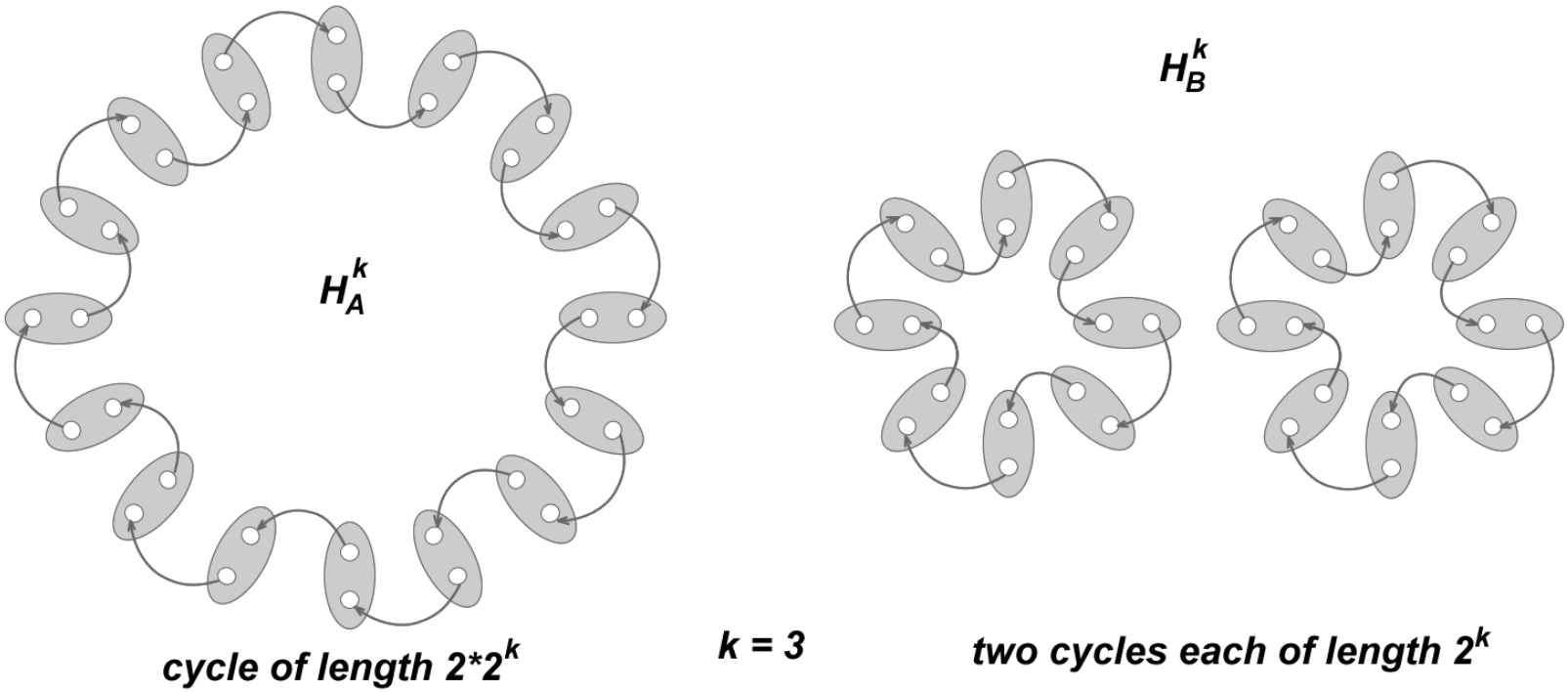
build two temporal graphs $\mathcal{H}_A^k$ and $\mathcal{H}_B^k$



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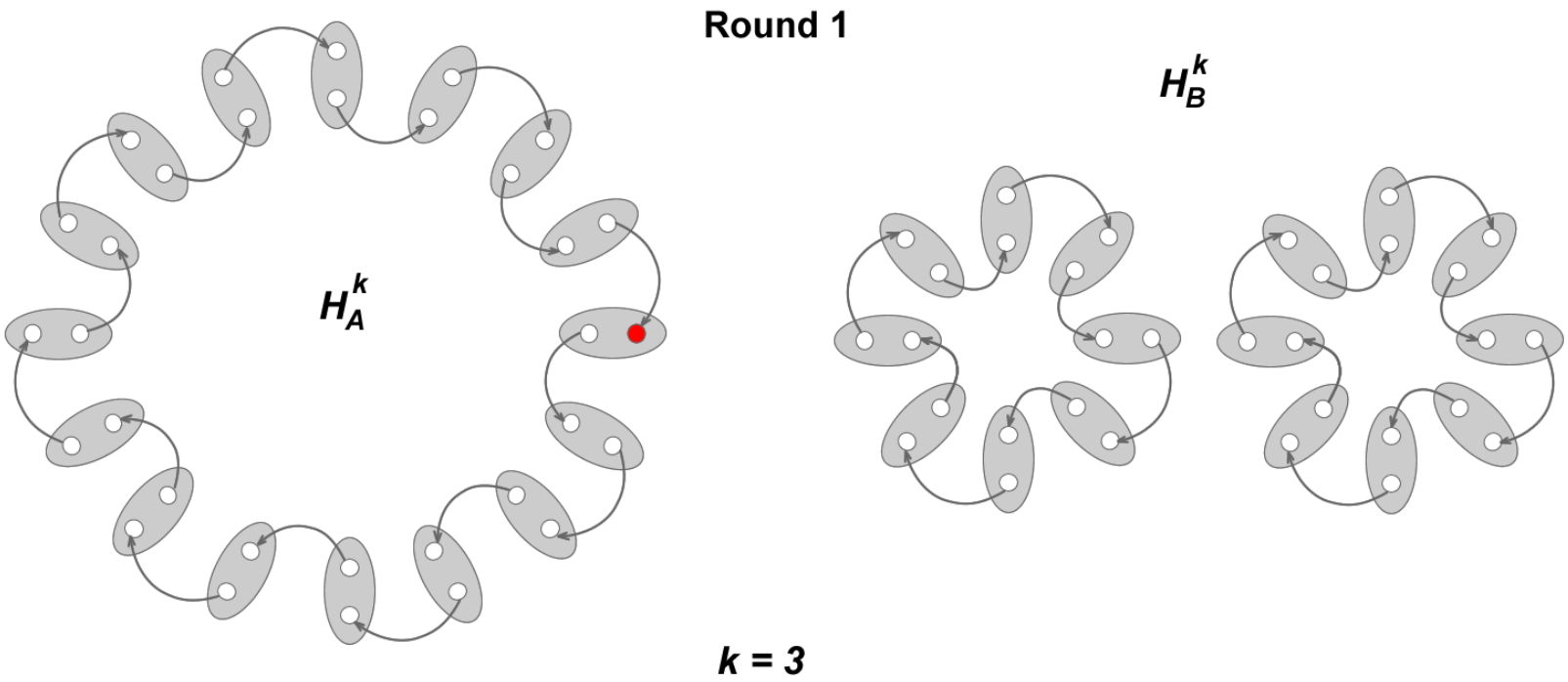

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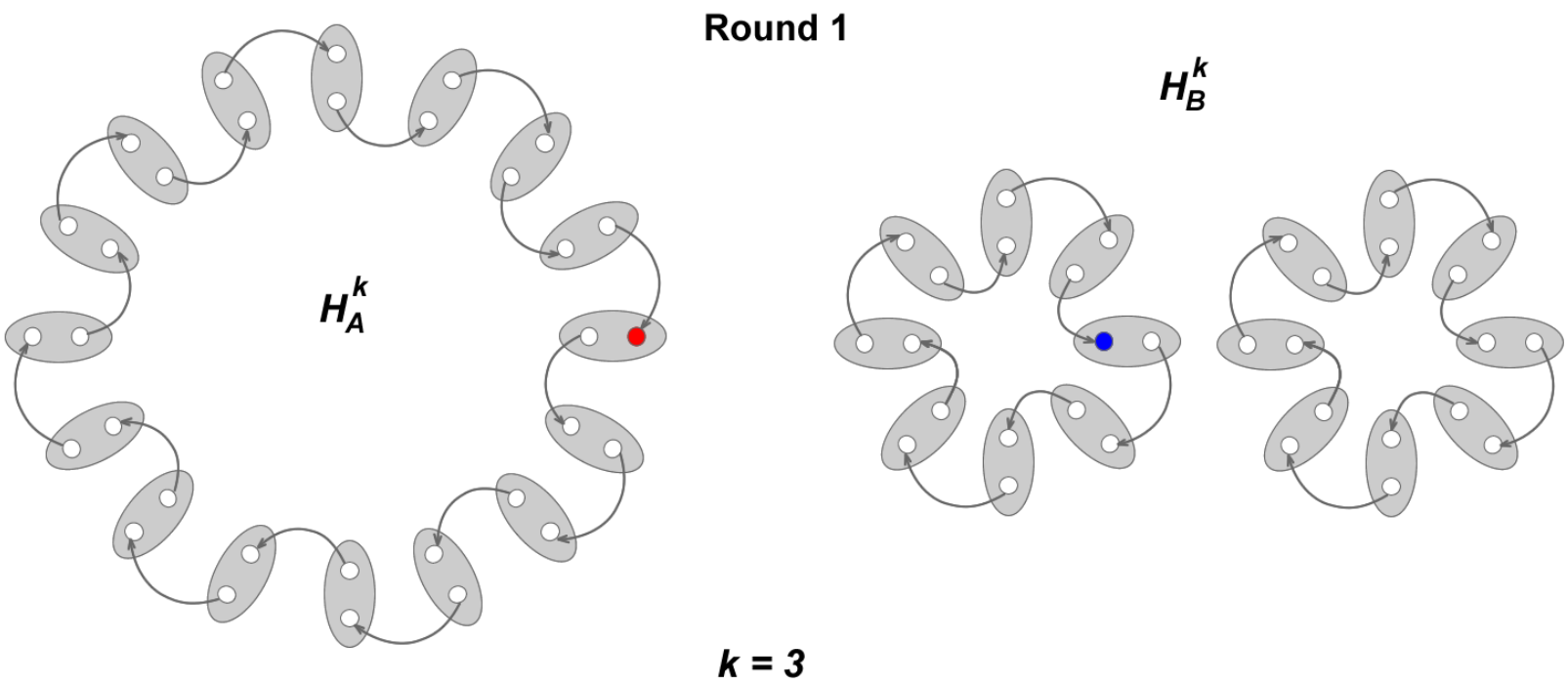

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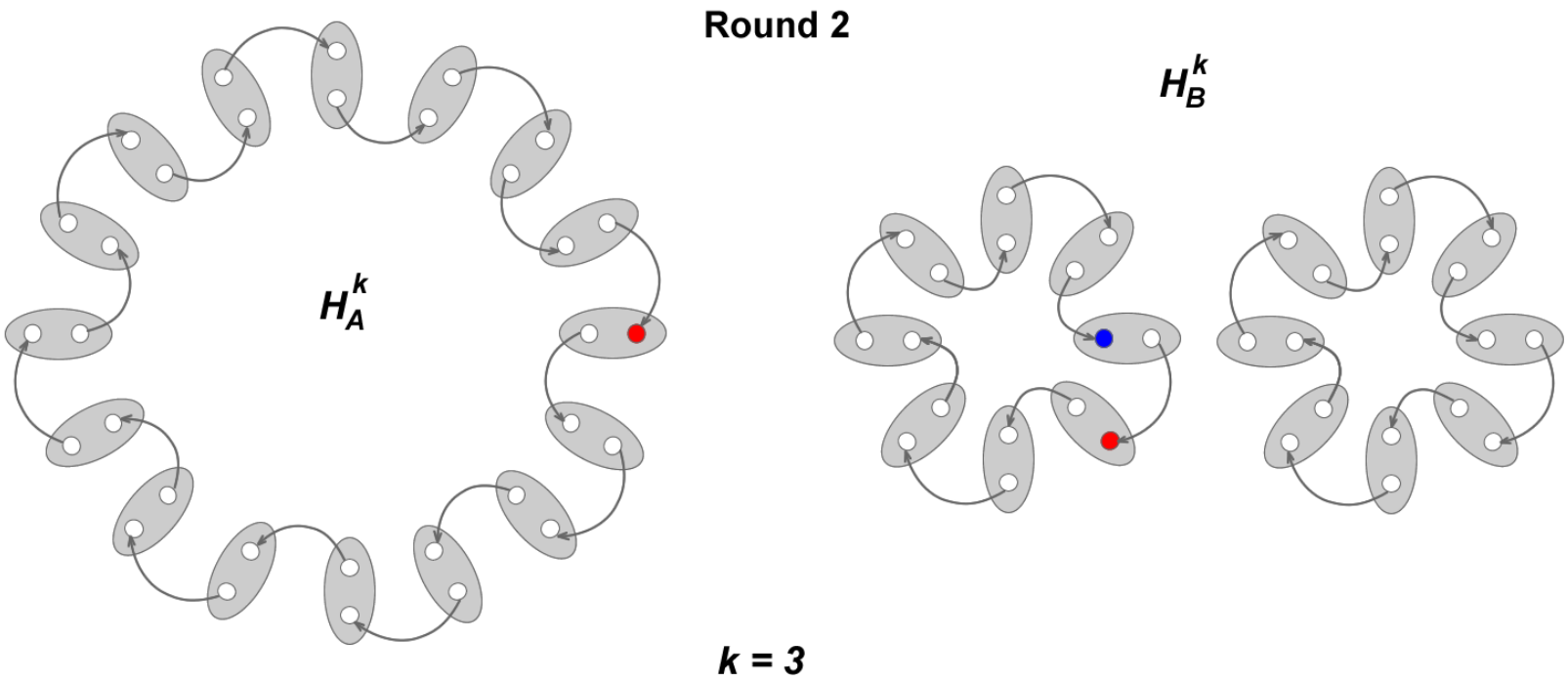

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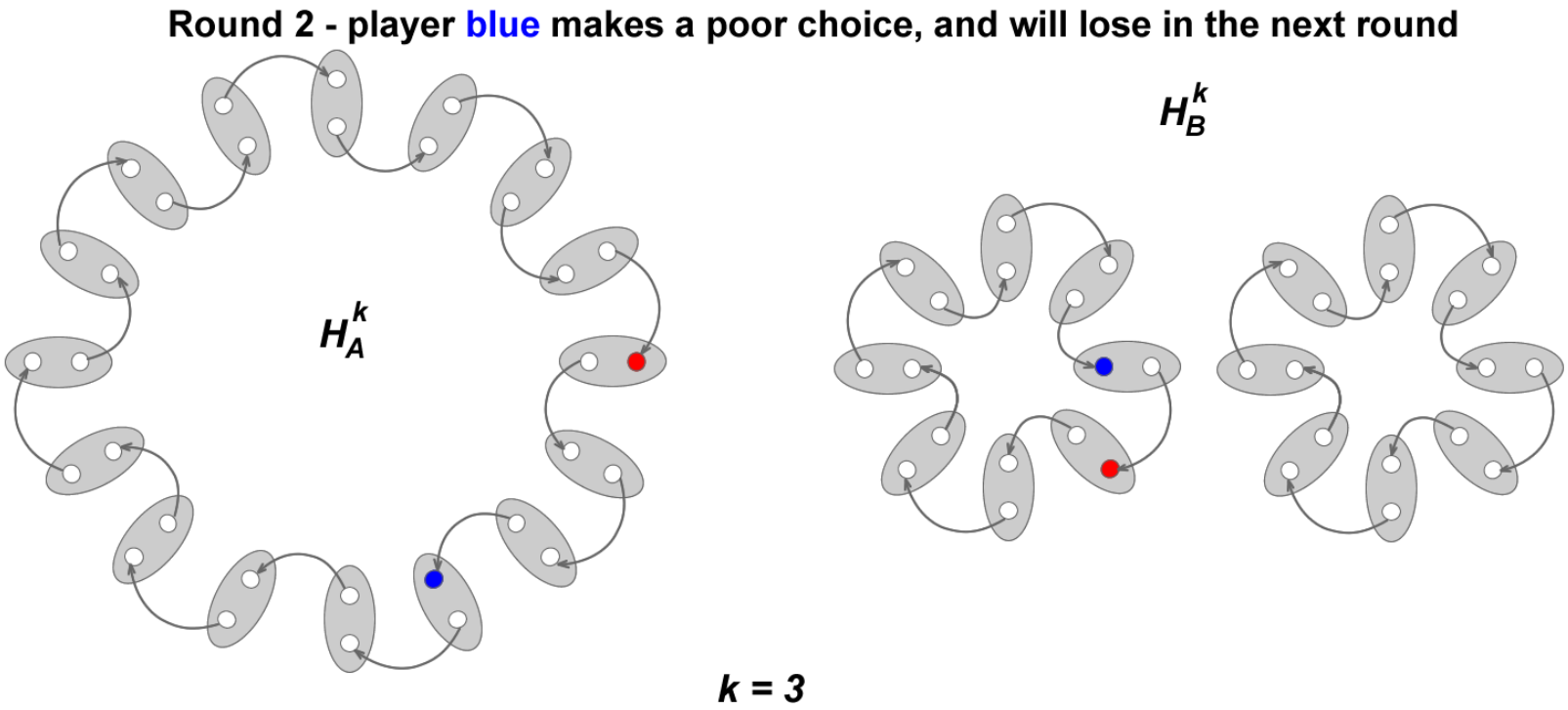

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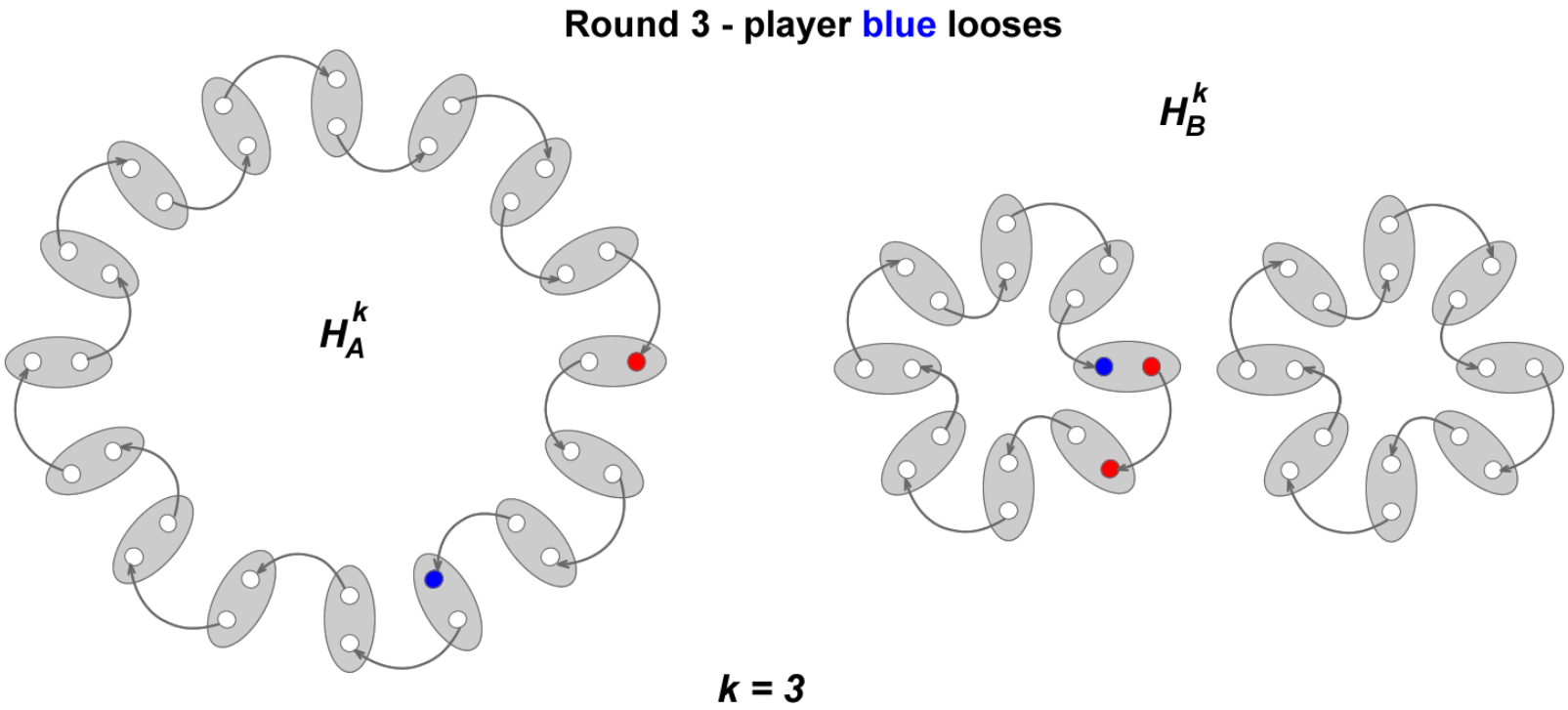

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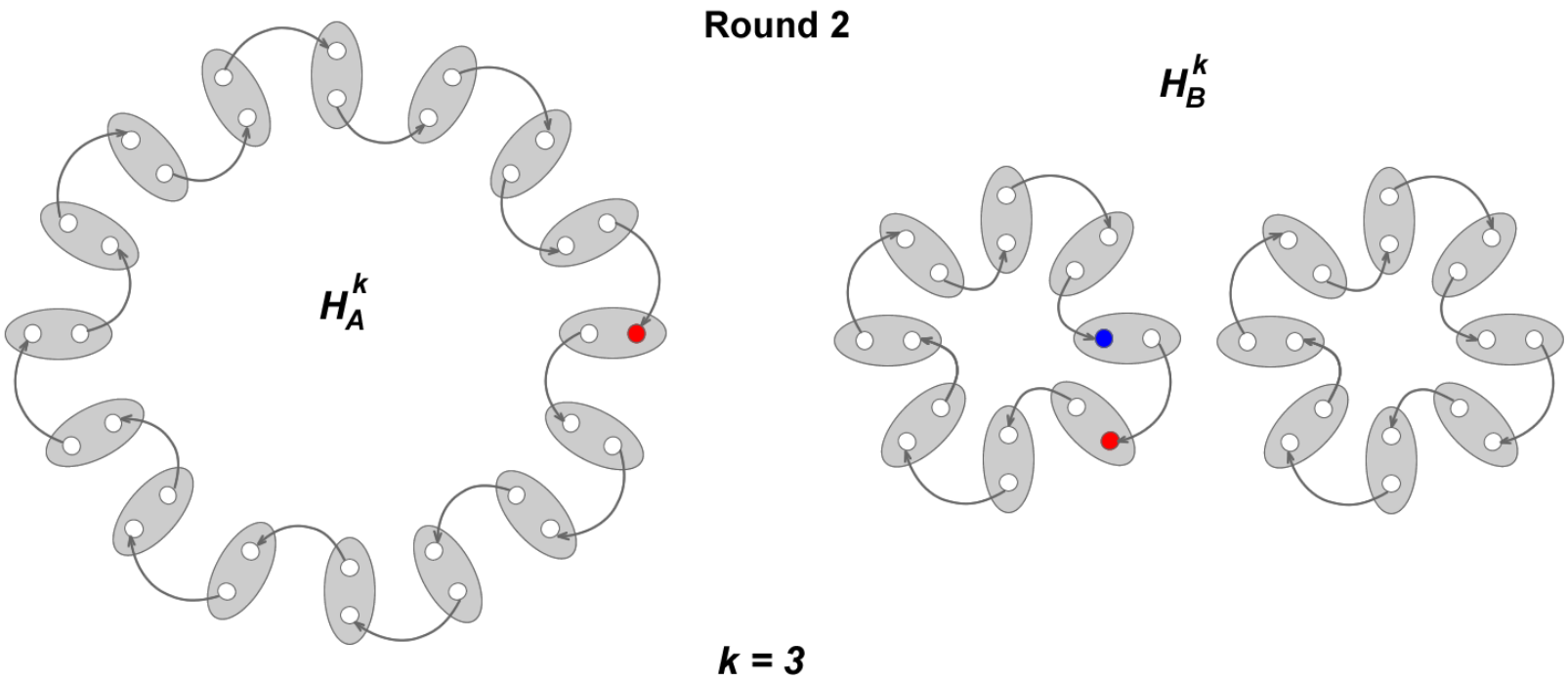

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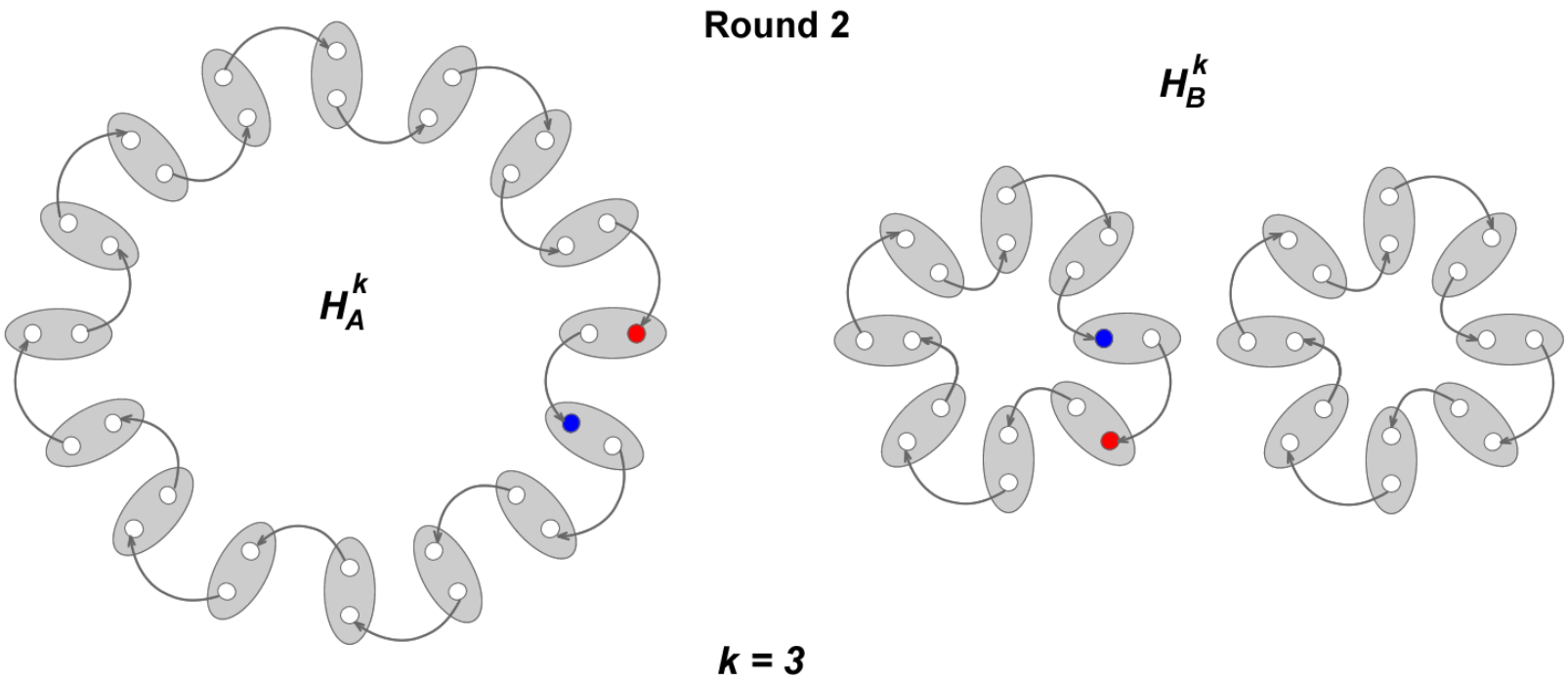

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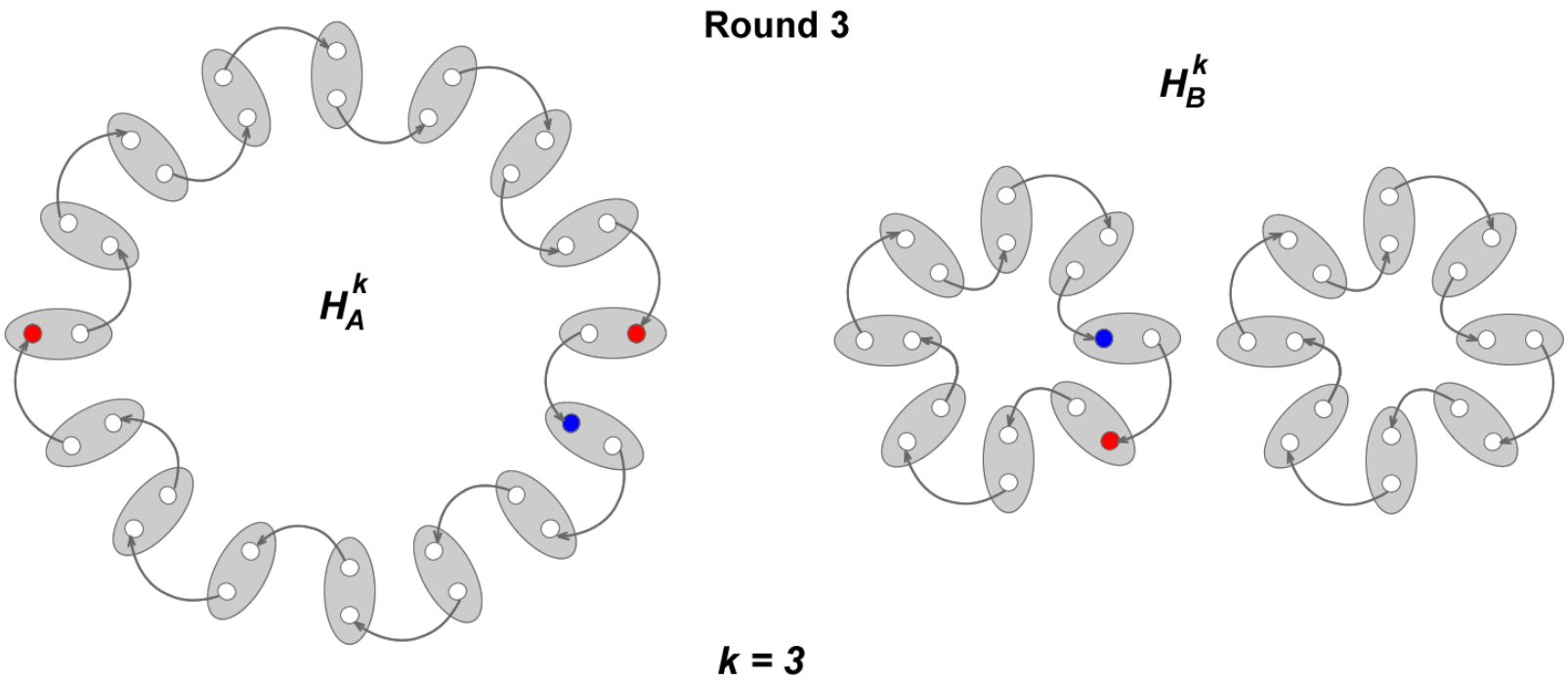

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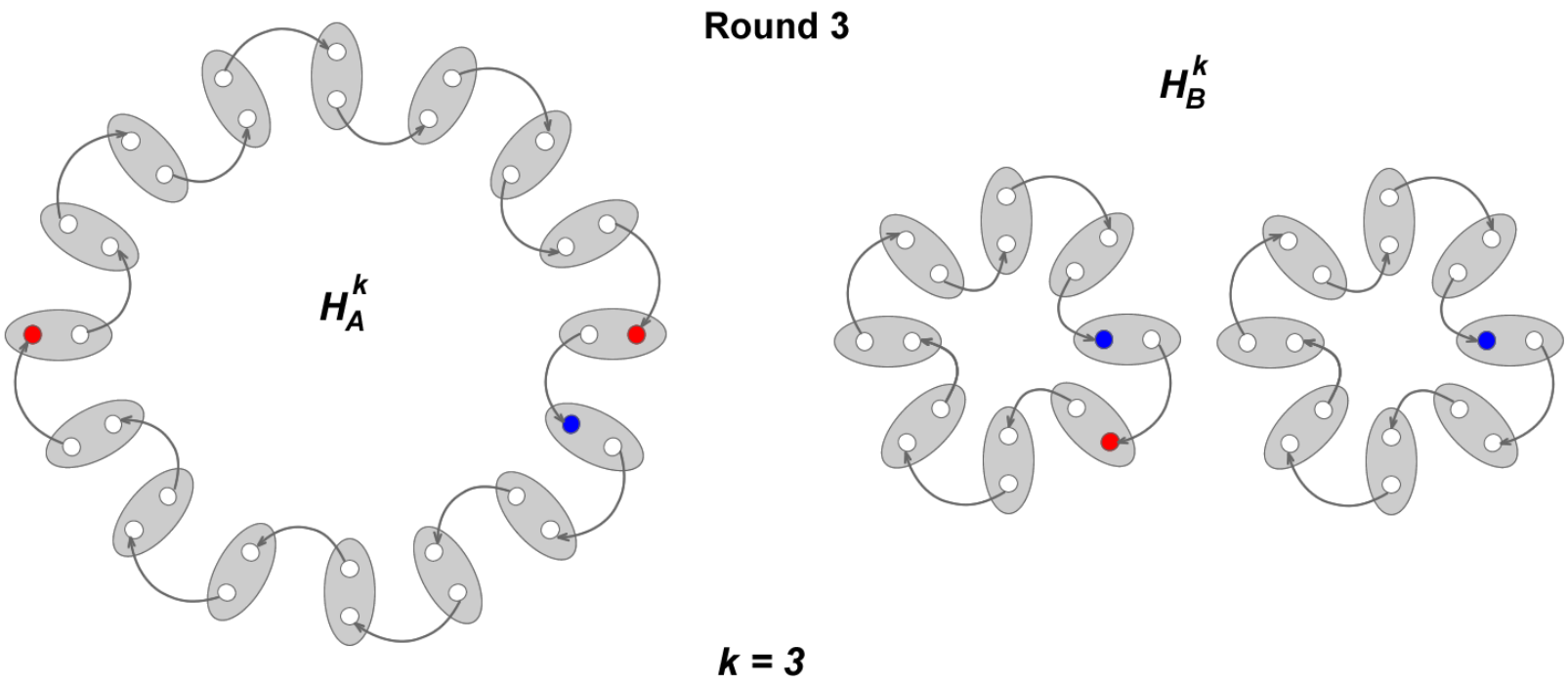

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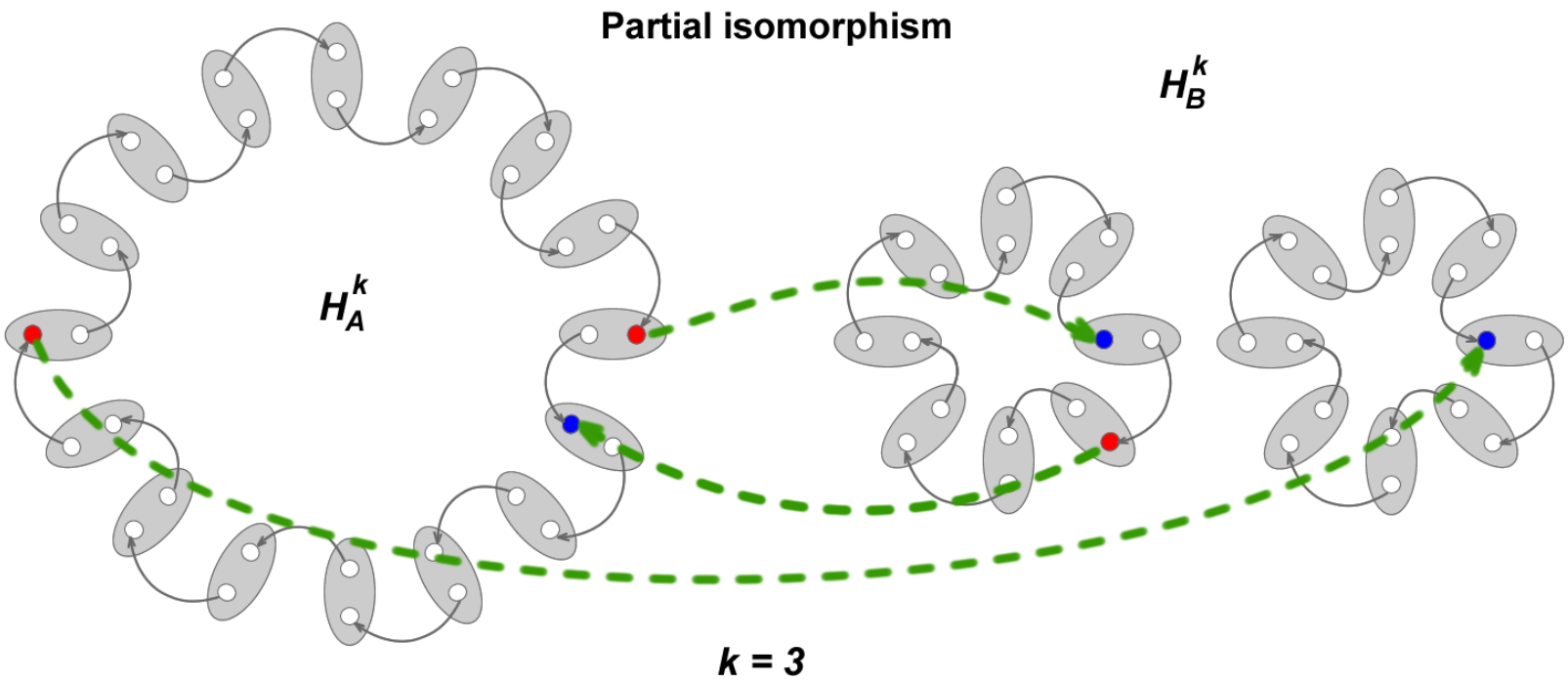

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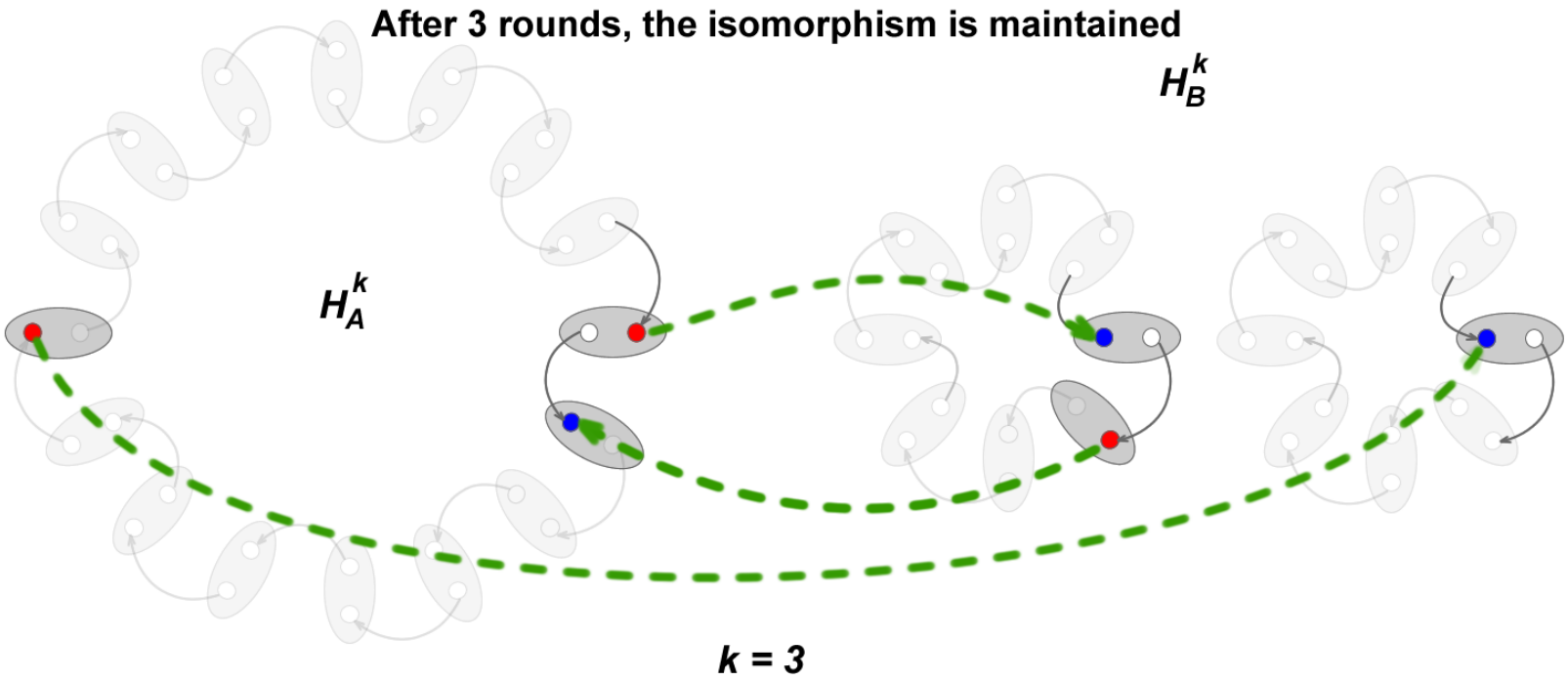

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Theorem (Ehrenfeucht-Fraïssé): If blue has a winning strategy on a k-**round** game, played over $\mathcal{H}_A^k$ and $\mathcal{H}_B^k$

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Theorem (Ehrenfeucht-Fraïssé): If blue has a winning strategy on a k-**round** game, played over $\mathcal{H}_A^k$ and $\mathcal{H}_B^k$ then there exists **no formula** from $FO[k]$ which is true in $\mathcal{H}_A^k$ and false $\mathcal{H}_B^k$

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Importance of our result:

- SQL **cannot** be used for querying about temporal graph properties
- Description Logics (without cyclic TBoxes) **cannot** be used to reason about temporal graphs
- A **language** which is suitable for reasoning about temporal graphs is *outside* FOL

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$L_{\mathcal{H}}$ Syntax

Let $\mathbb{V}$ars be a set of variables, $\mathcal{A}$ be a $\sigma_E \cup \sigma_A$-structure and $\mathcal{H}_{\mathcal{A}}$ be a $\mathcal{A}$-labelled **t**-graph. Also, let $X \in \{E, A\}$, and $\dagger_E \in \mathfrak{C}_Q$ and $\dagger_A \in \mathfrak{C}_A$. The syntax of a X-formula is recursively defined with respect to $\mathcal{A}$, as follows:

- if $R \in \sigma_X$ with arity$(R) = n$ and $\bar{t} \in (\mathbb{V}\text{ars} \cup I)^n$, then $R(\bar{t})$ is an **atomic** Q-formula (or an atom).
- if ϕ is a X-formula then (ϕ) is also a X-formula;
- if ϕ, ψ are X-formulae then $\phi \dagger_X \psi$ and $\phi \neg \dagger_X \psi$ are also X-formulae. We call $\dagger_X$ a **positive** relation and $\neg\dagger_X$ a **negative/negated** relation.
- Let $R \in \sigma_X$, ϕ, ψ, ω be X-formulae and $\dagger_X, \dagger'_X$ designate positive or negated relations. If ϕ has any of the following forms: (i) $\phi = R(\bar{t})$, (ii) $\phi = R(\bar{t}) \dagger_X \psi$ or (iii) $\phi = (R(\bar{t}) \dagger_X \omega) \dagger'_X \psi$, then it is R**-compatible**.

If ϕ and φ are both R**-compatible**, then $\phi \wedge \varphi$ and $\phi \vee \varphi$ are X formulae, and R**-compatible** as well.

If ϕ is a E or A-formula and $\overline{x}$ are variables occurring in ϕ, then we also write $\phi(\overline{x})$, to highlight these variables.

$L_{\mathcal{H}}$ Semantics

Let $\mathcal{A}$ be a $\sigma_E \cup \sigma_A$ (labelling) structure, and $\mathcal{H}$ be a $\mathcal{A}$-labelled **t**-graph. $\bar{i} \in \mathcal{A}$, $R \in \sigma_E \cup \sigma_A$, a, a' denote action nodes from $\mathcal{H}$, and q, q' denote quality edges. We write $\phi_{[\overline{x}\setminus\bar{i}]}$, to refer to the formula obtained from ϕ by replacing all variables from $\overline{x}$ with individuals from $\bar{i}$. Also, let $X \in \{E, A\}$. The semantics of X-formulae is defined as follows:

1. $\|\phi(\overline{x})\|^X_{\mathcal{H}} = \bigcup_{\bar{i}} \{q \in \|\phi_{[\overline{x}\setminus\bar{i}]}\|^Q_{\mathcal{H}}\}$;
2. $\|R(\bar{i})\|^X_{\mathcal{H}} = \{q : R(\bar{i}) \in \mathcal{L}_X(q)\}$;
3. $\|(\phi)\dagger_Q \psi\|^Q_{\mathcal{H}} = \{q \in \|\phi\|^Q_{\mathcal{H}} : \exists q' \in \|\psi\|^Q_{\mathcal{H}}$ such that $\lambda_{\dagger_Q}(q, q')\}$
4. $\|(\phi)\dagger_A \psi\|^A_{\mathcal{H}} = \{a \in \|\phi\|^A_{\mathcal{H}} : \exists a' \in \|\psi\|^A_{\mathcal{H}}$ such that $\lambda_{\dagger_A}(a, a')\}$
5. $\|\phi\neg \dagger_X \psi\|^X_{\mathcal{H}} = \|(\phi)\neg \dagger_X \psi\|^X_{\mathcal{H}} = \|\phi\|^X_{\mathcal{H}} \setminus \|\phi \dagger_X \psi\|^X_{\mathcal{H}}$;
6. $\|\phi \dagger_X (\psi)\|^X_{\mathcal{H}} = \|\phi \dagger_X \psi\|^X_{\mathcal{H}}$
7. $\|R(\bar{i}) \dagger_X \psi\|^X_{\mathcal{H}} = \|(R(\bar{i})) \dagger_X \psi\|^X_{\mathcal{H}}$;
8. $\|\phi \vee \psi\|^X_{\mathcal{H}} = \|\phi\|^X_{\mathcal{H}} \cup \|\psi\|^X_{\mathcal{H}}$;
9. $\|\phi \wedge \psi\|^X_{\mathcal{H}} = \|\phi\|^X_{\mathcal{H}} \cap \|\psi\|^X_{\mathcal{H}}$;

The language $L_{\mathcal{H}}$

Let $\varphi \in L_{\mathcal{H}}$ be a formula

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Let $\varphi \in L_{\mathcal{H}}$ be a formula and $\mathcal{H}$ be a temporal graph.

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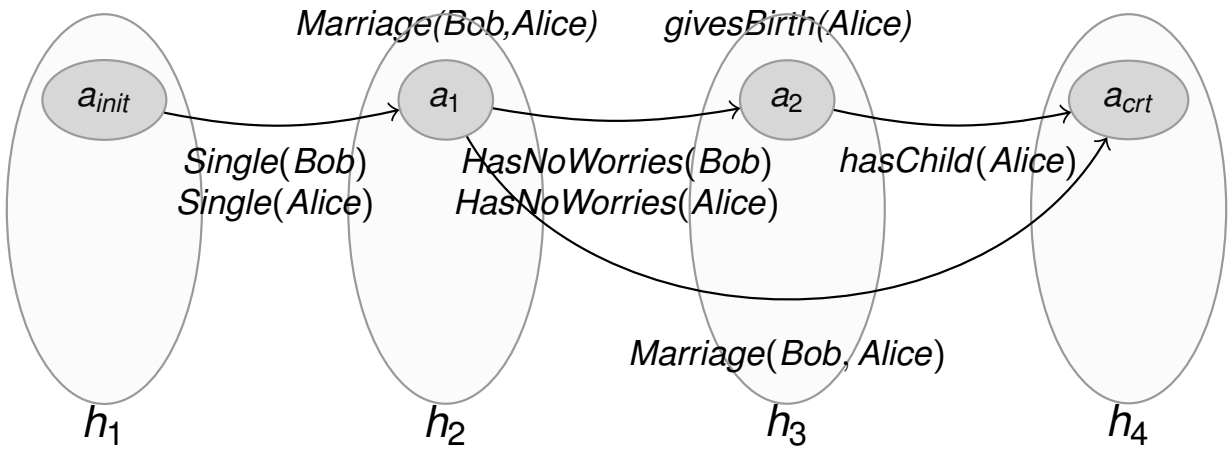

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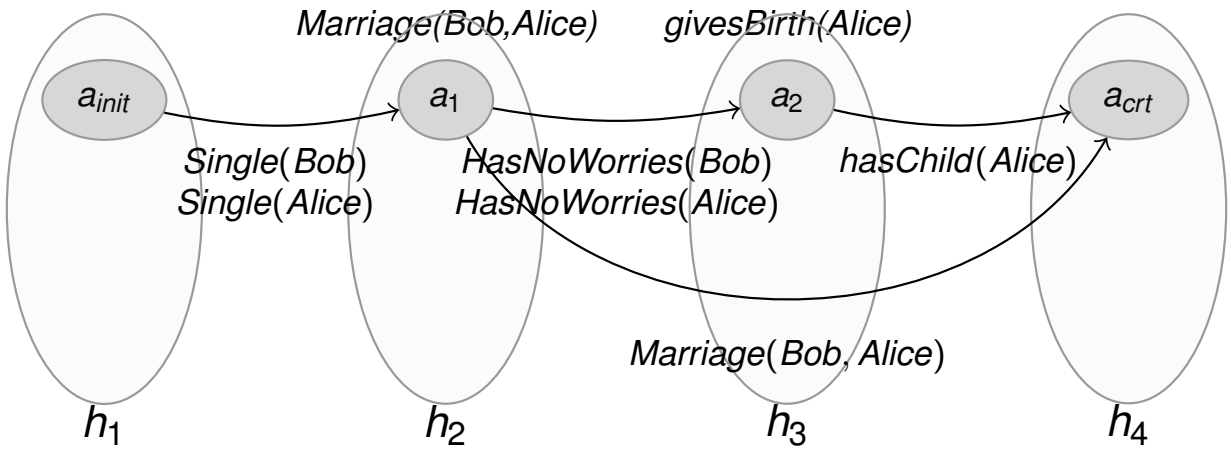

$\|\text{Single(x)}\|^E_{\mathcal{H}}$

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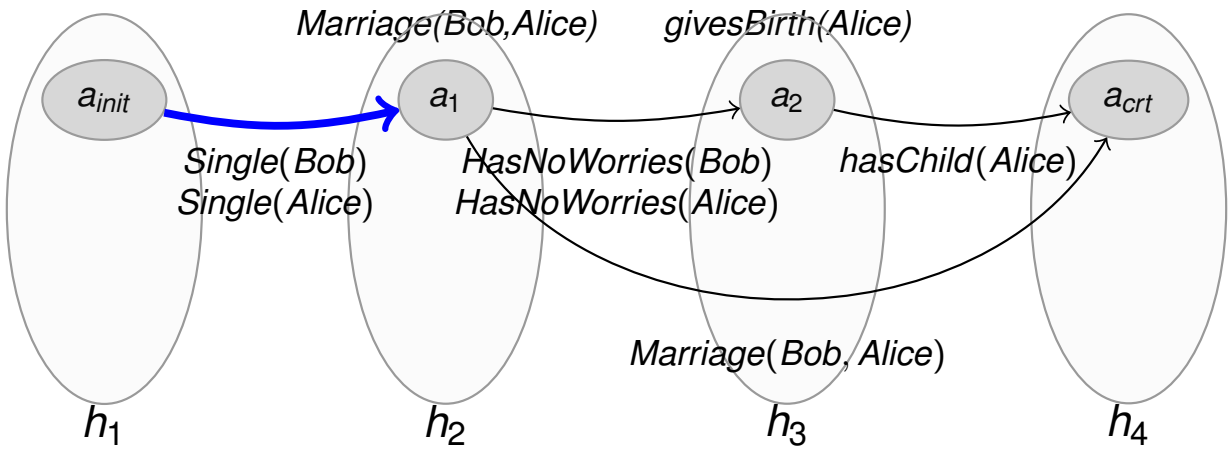

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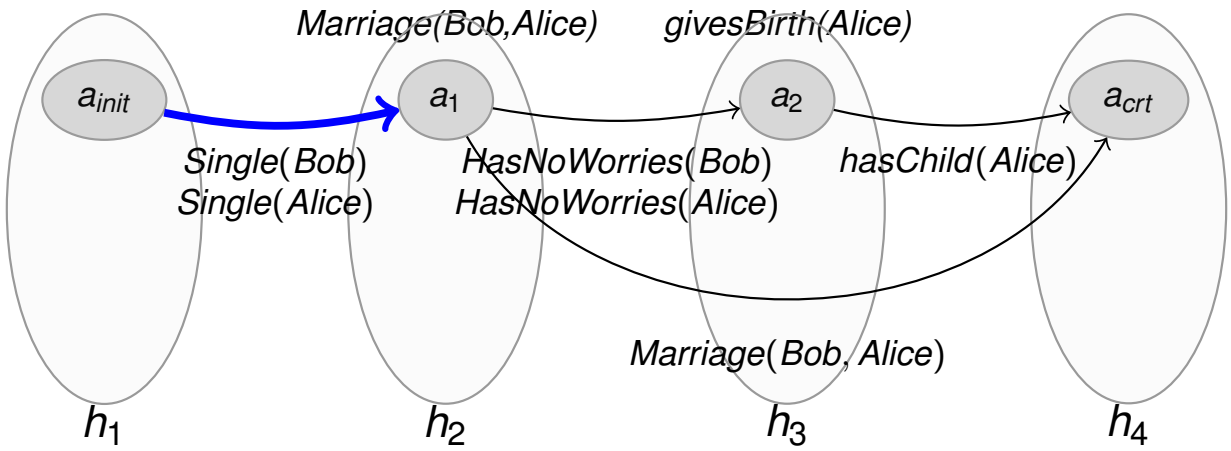

$\|\text{Single(x)}\|^{E}_{\mathcal{H}} = \{(a_{init}, a_1)\}$

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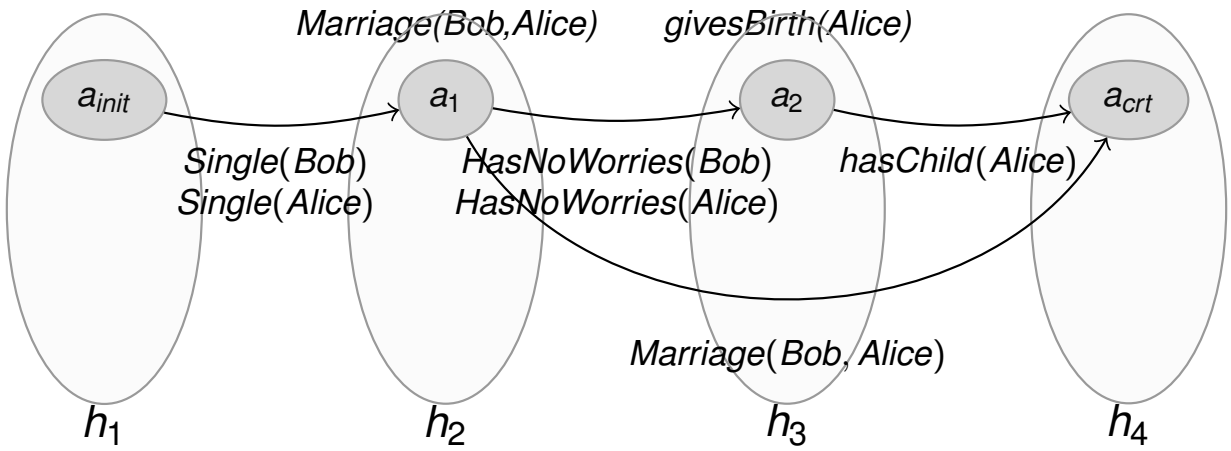

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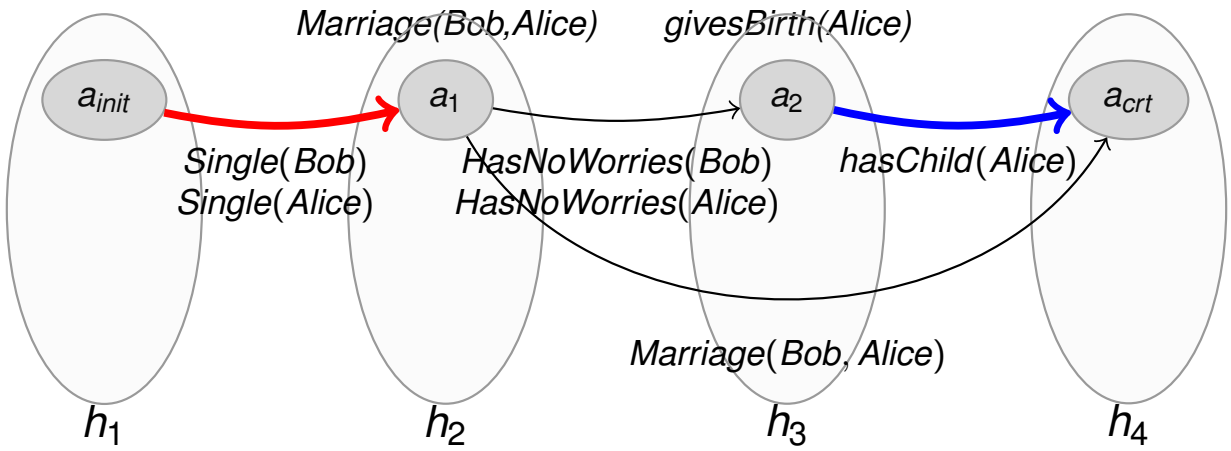

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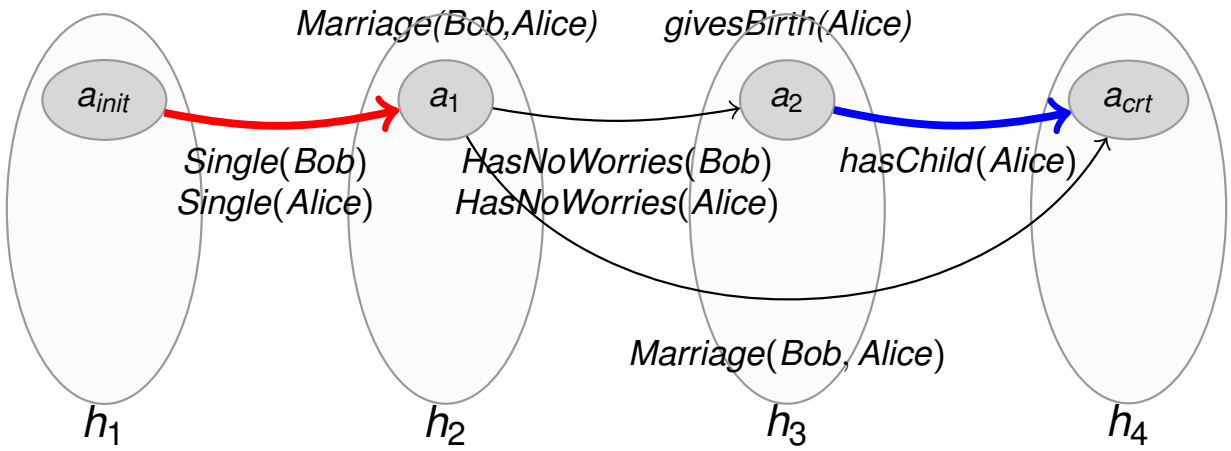

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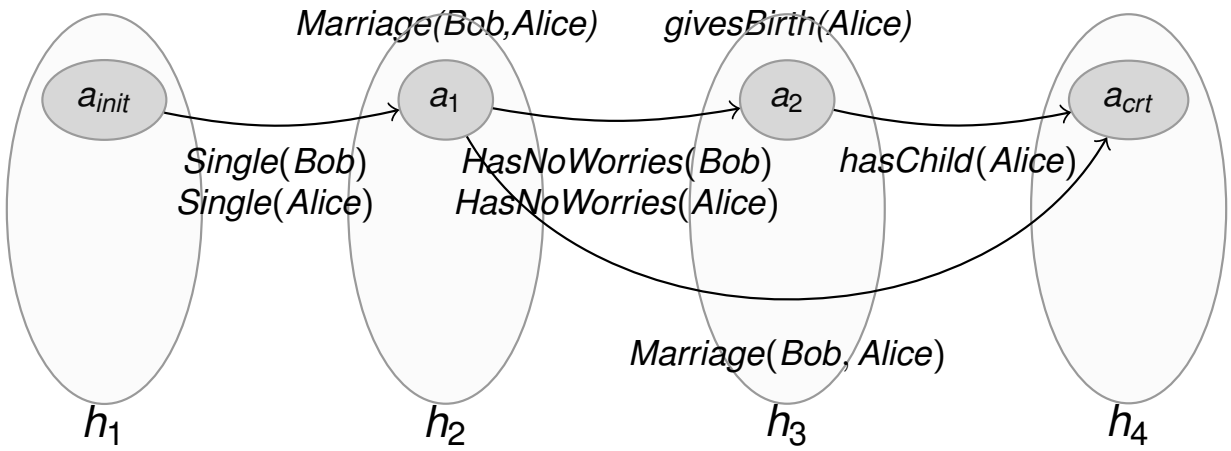

$\|\mathit{HasNoWorries}(x)$ **after** $\mathit{Single}(x) \wedge$
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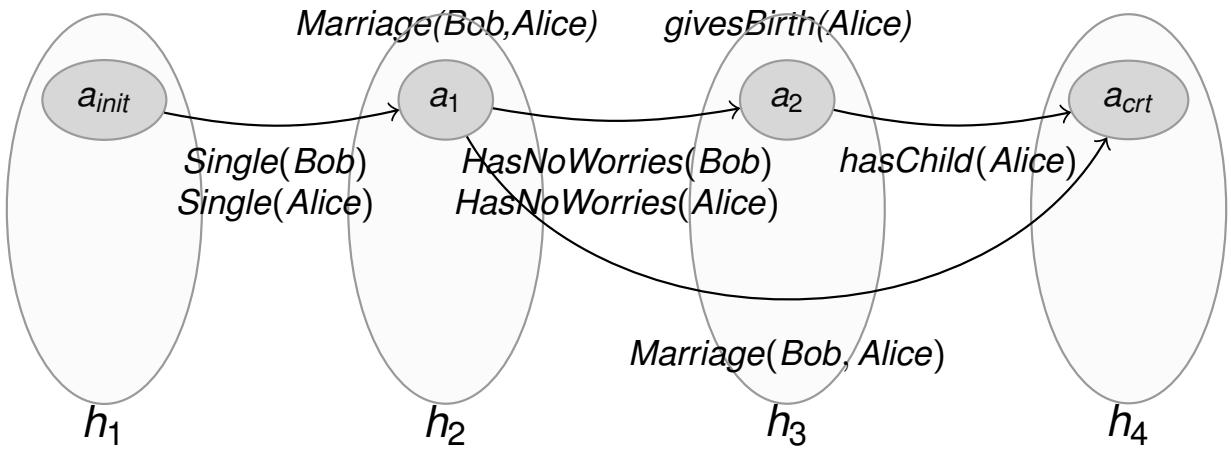

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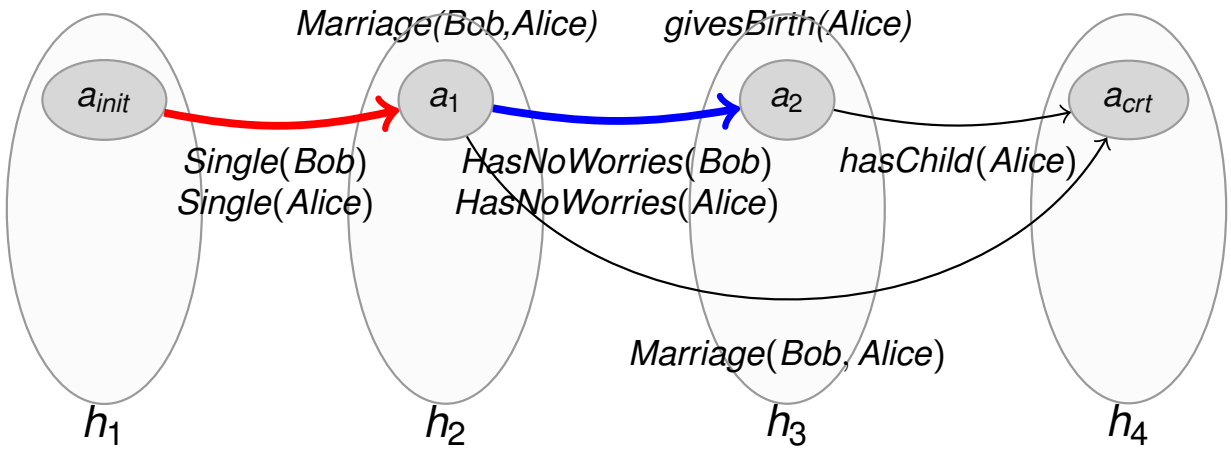

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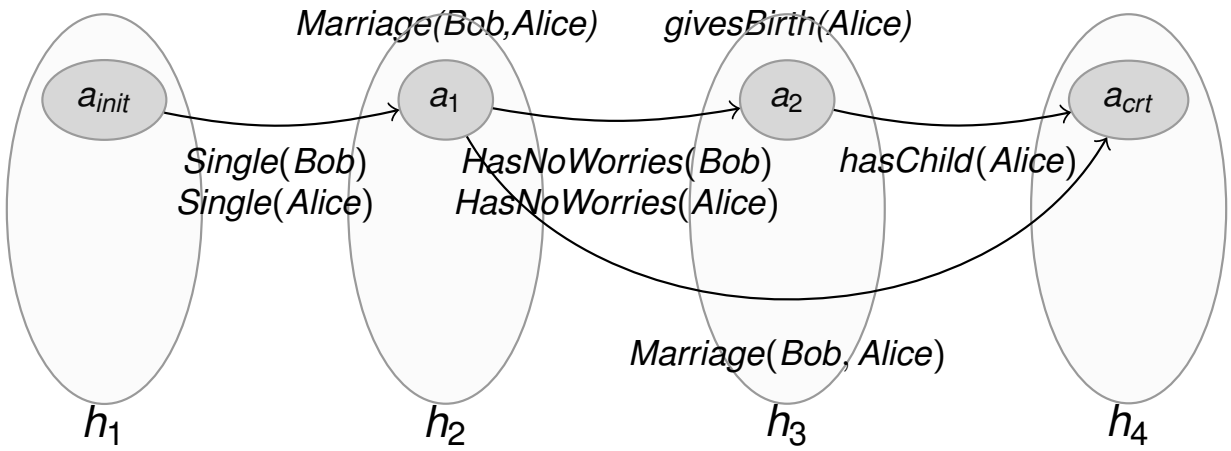

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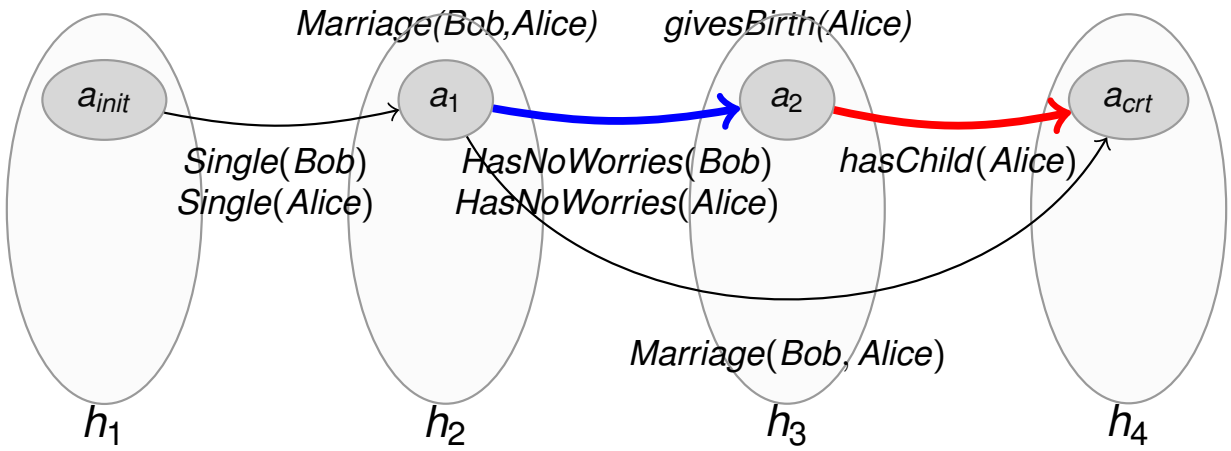

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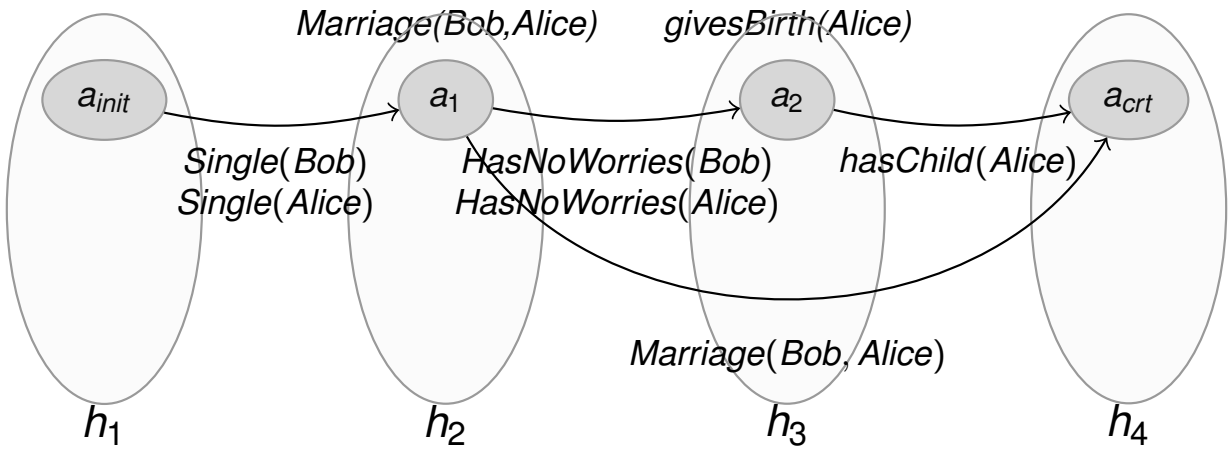

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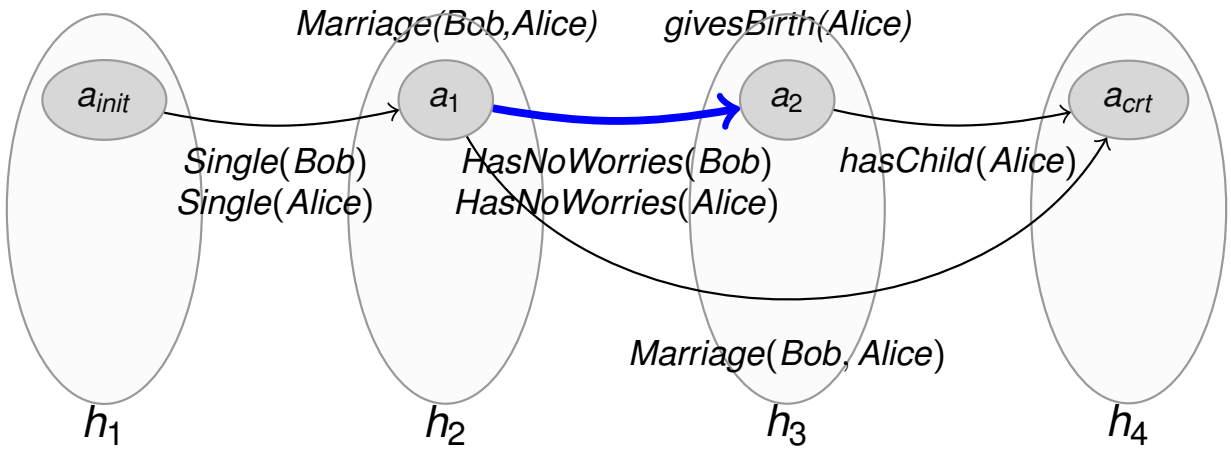

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Outline

Complexity results

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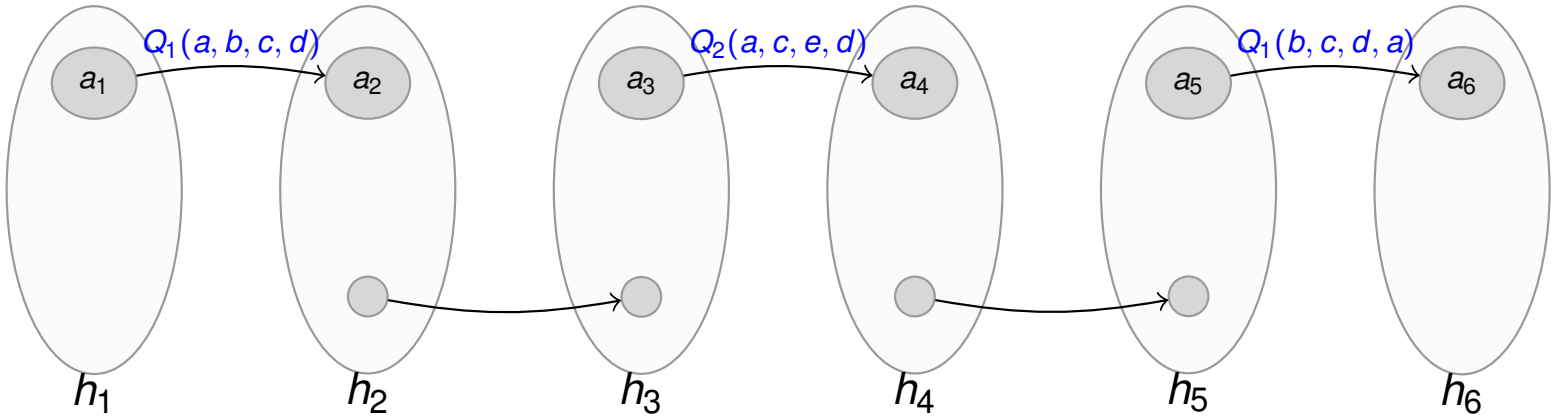

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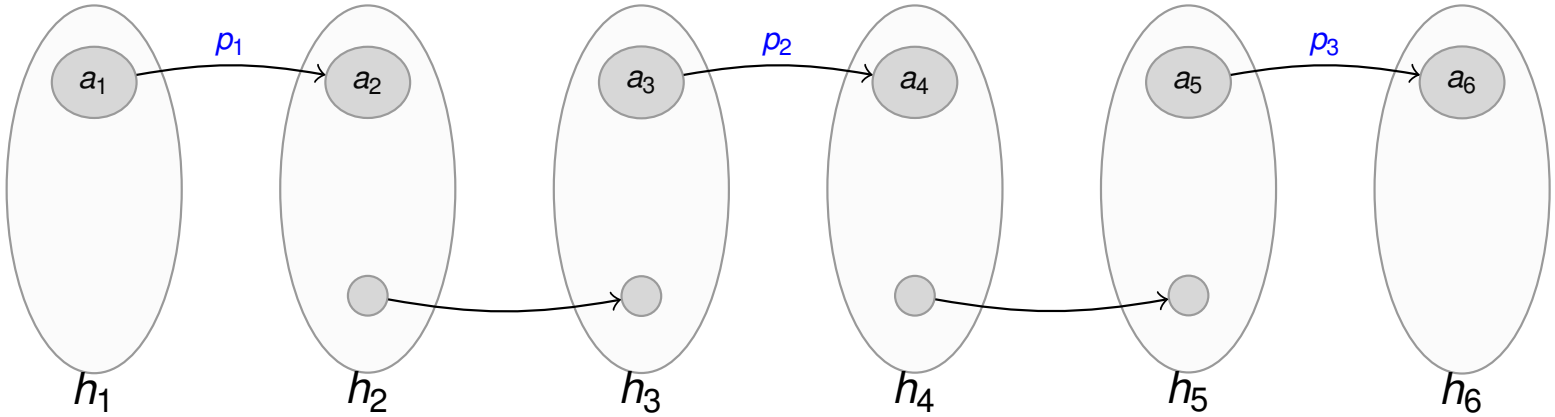

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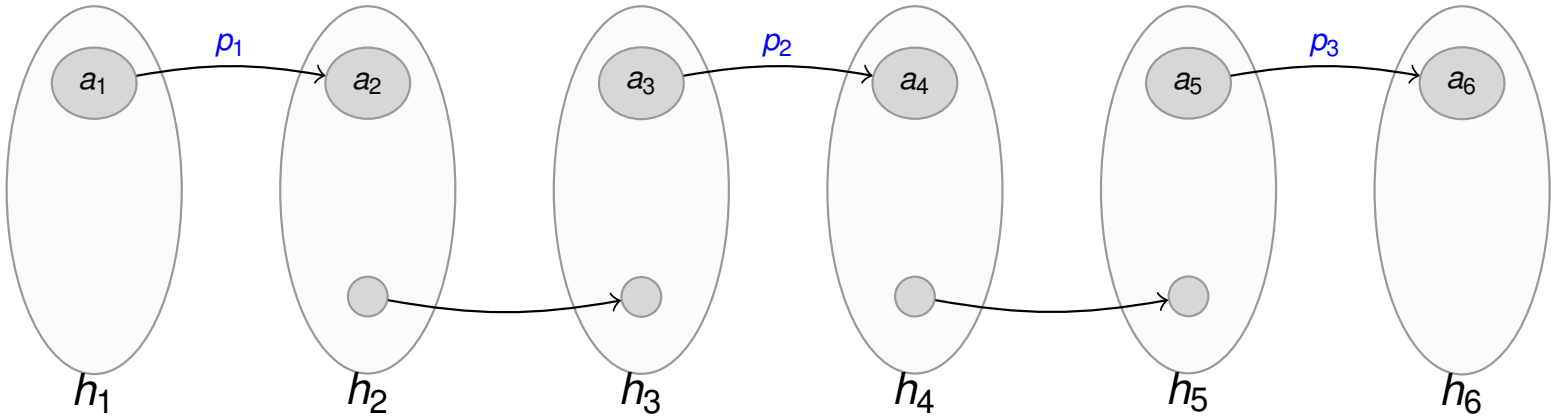

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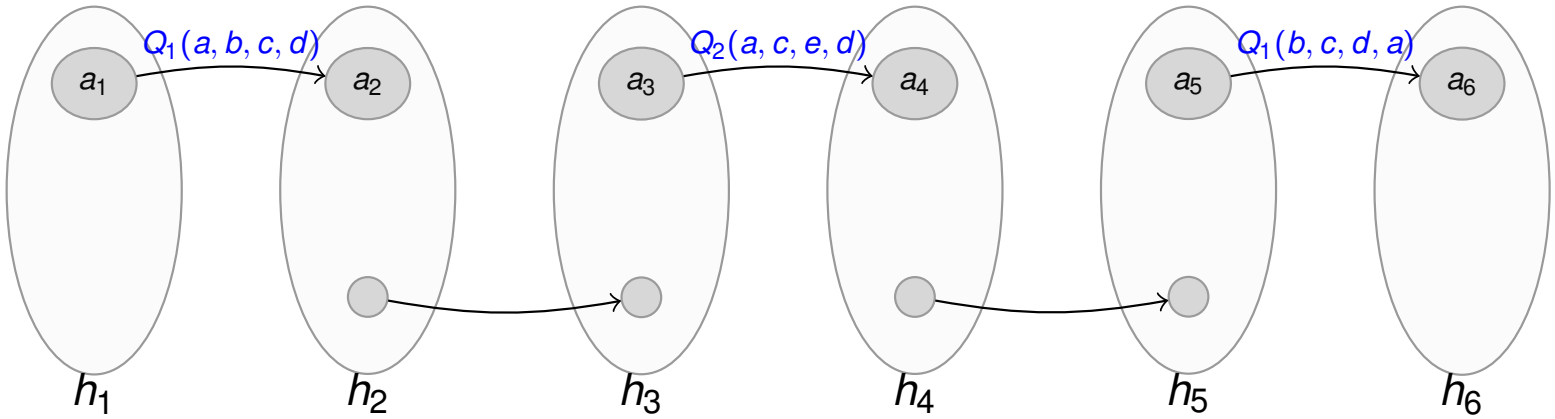

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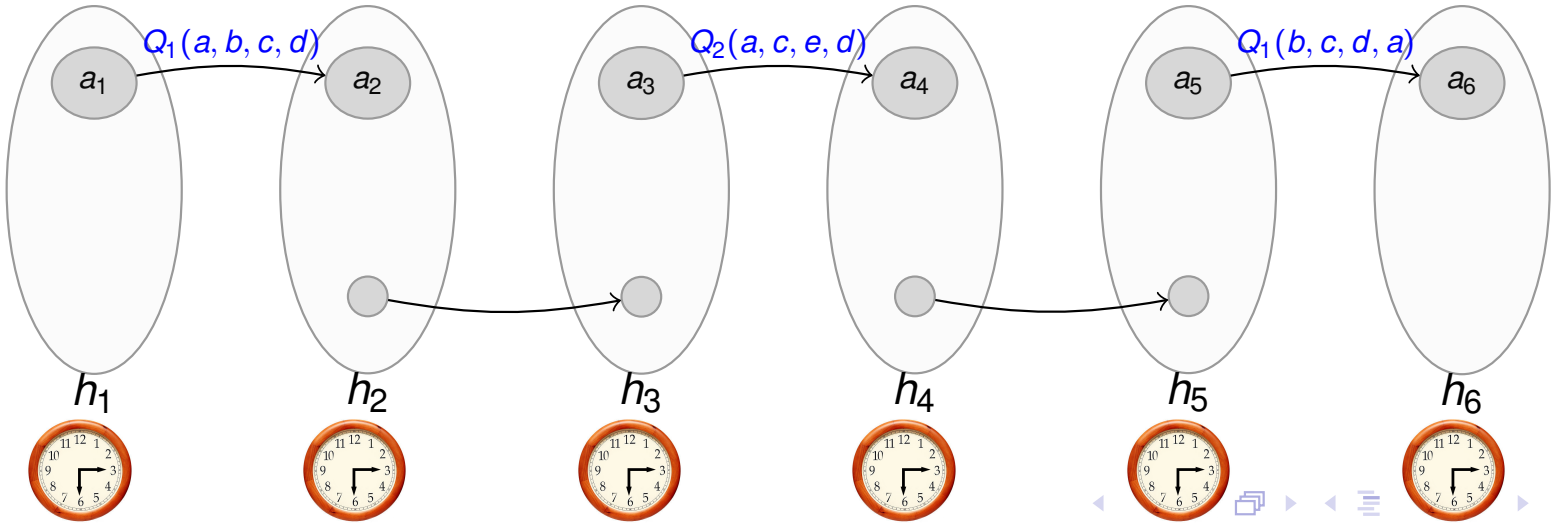

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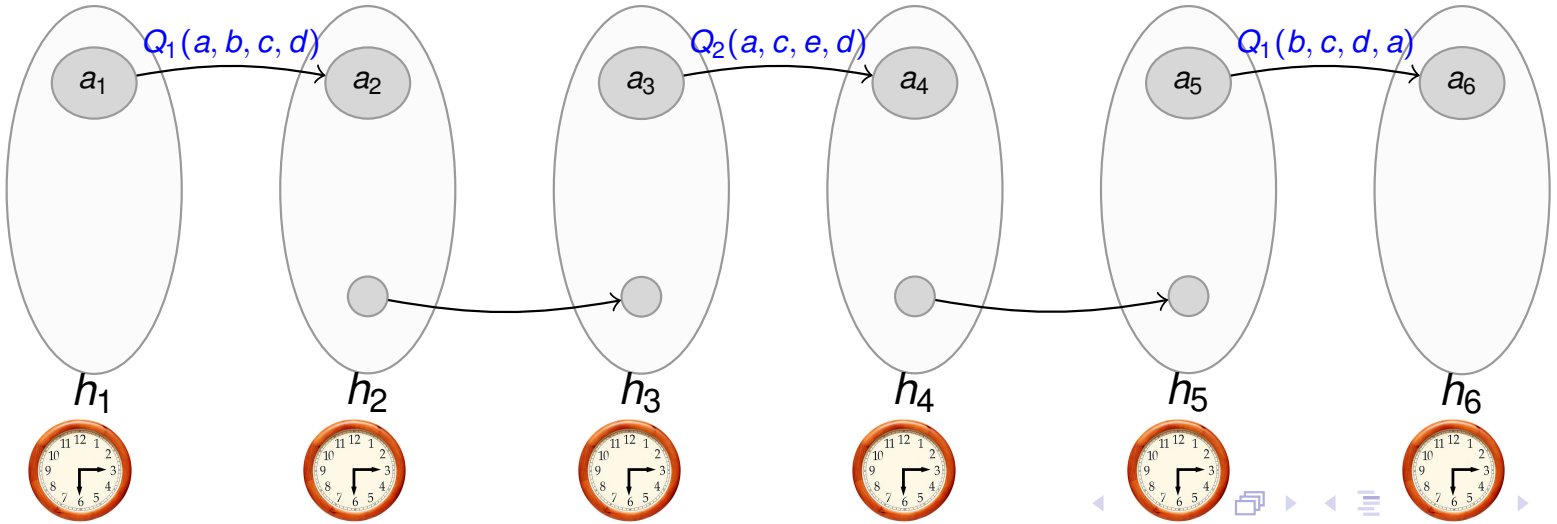

Complexity results

Comparing $L_{\mathcal{H}}$ model checking with other approaches:

Method	**Reasoning problem**	**Complexity**
The TEDL $\mathcal{HS}$	satisfiability	*undecidable*
The TEDL $\mathcal{ALCT}$	satisfiability	*PSPACE – complete*
The DL $\mathcal{ALC}$(cyclic TBoxes)	subsumption	*EXPTIME*
The DL $\mathcal{FL}_0$(cyclic TBoxes)	subsumption	*PSPACE – complete*
$L_{\mathcal{H}}$	model-checking	*NP – complete*
$L^*_{\mathcal{H}}$	model-checking	*PTIME*

Applications of temporal graphs and $L_{\mathcal{H}}$

- Specifying **time-dependent** device behaviour in **intelligent buildings**
- Data mining logs from HPC systems
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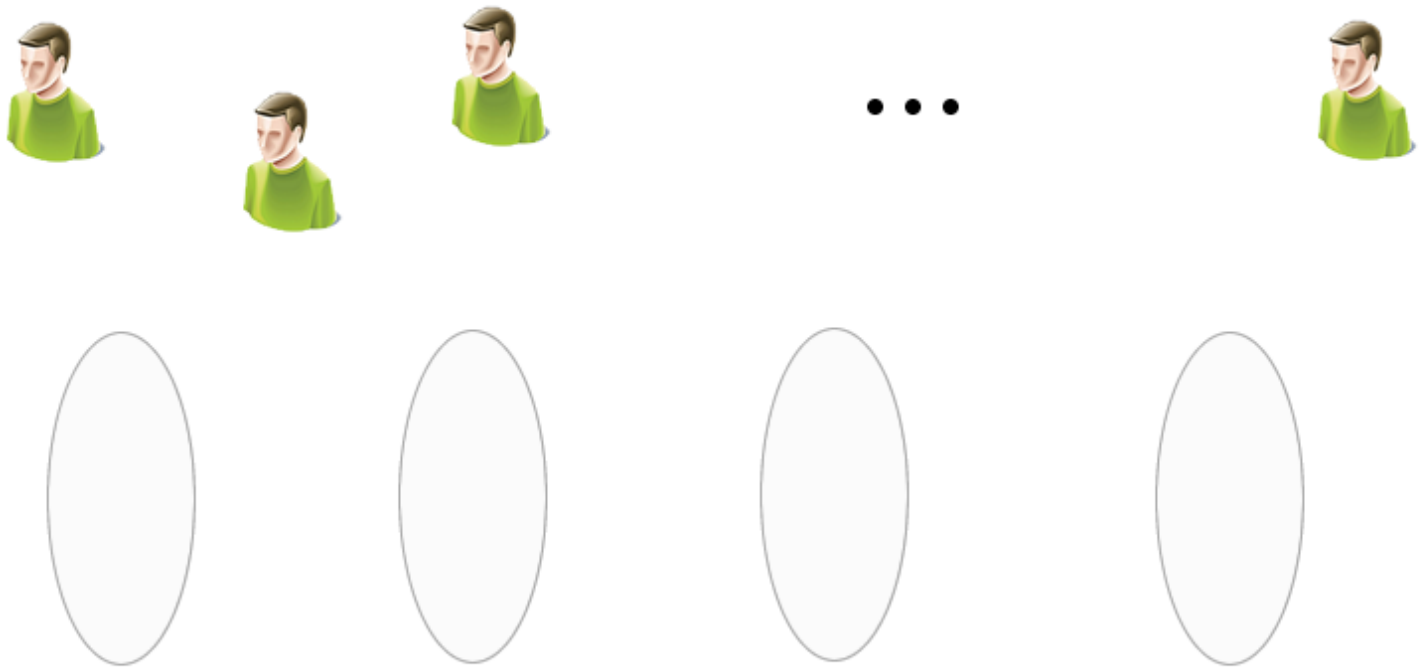
Temporal graphs and $L^*_{\mathcal{H}}$ in MAS

Setting: a finite set of **players**

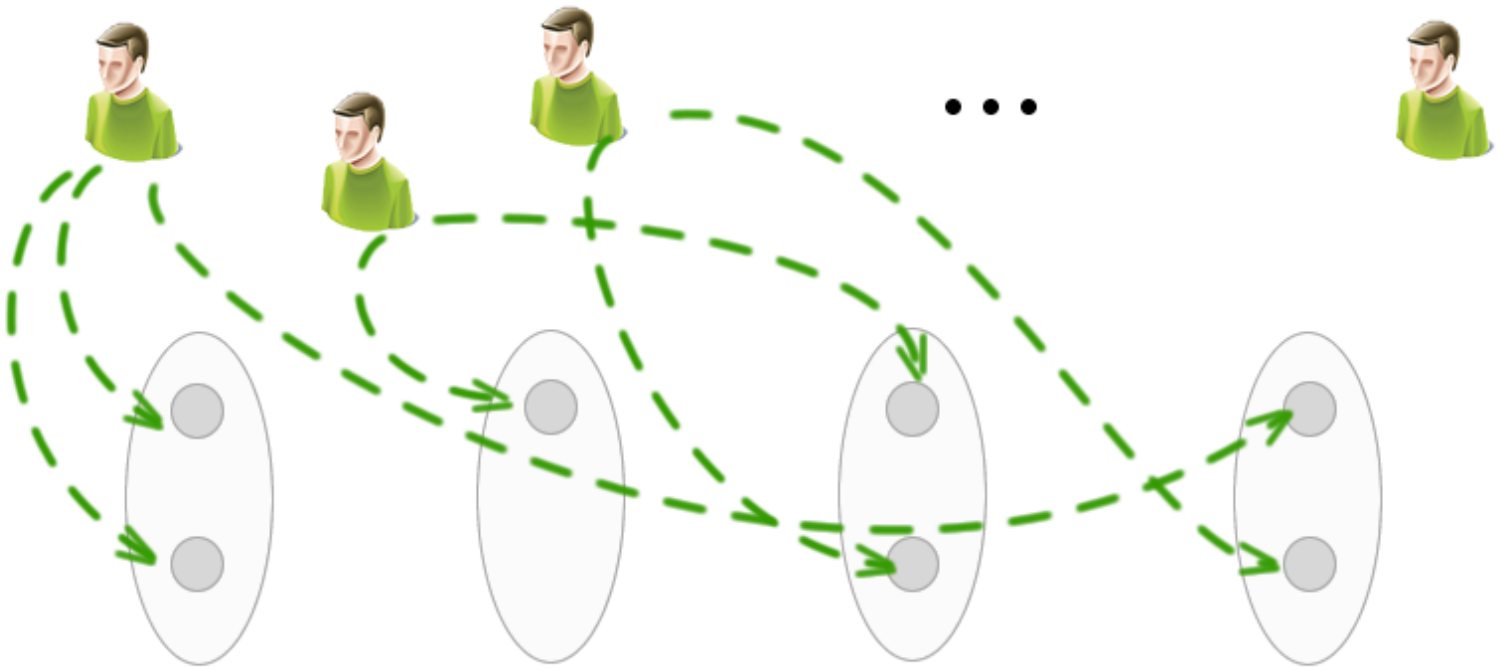
Temporal graphs and $L^*_{\mathcal{H}}$ in MAS

a finite set of **hypernodes**



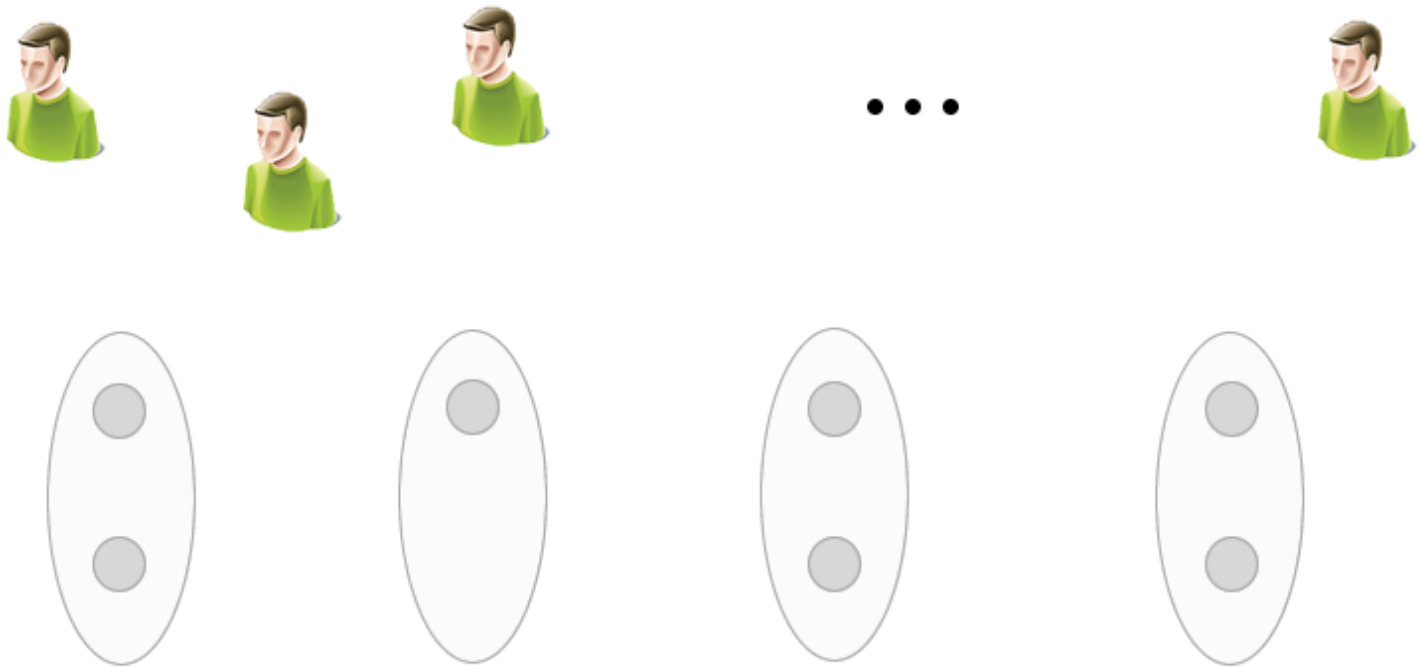
Temporal graphs and $L^*_{\mathcal{H}}$ in MAS

possible actions, for each player



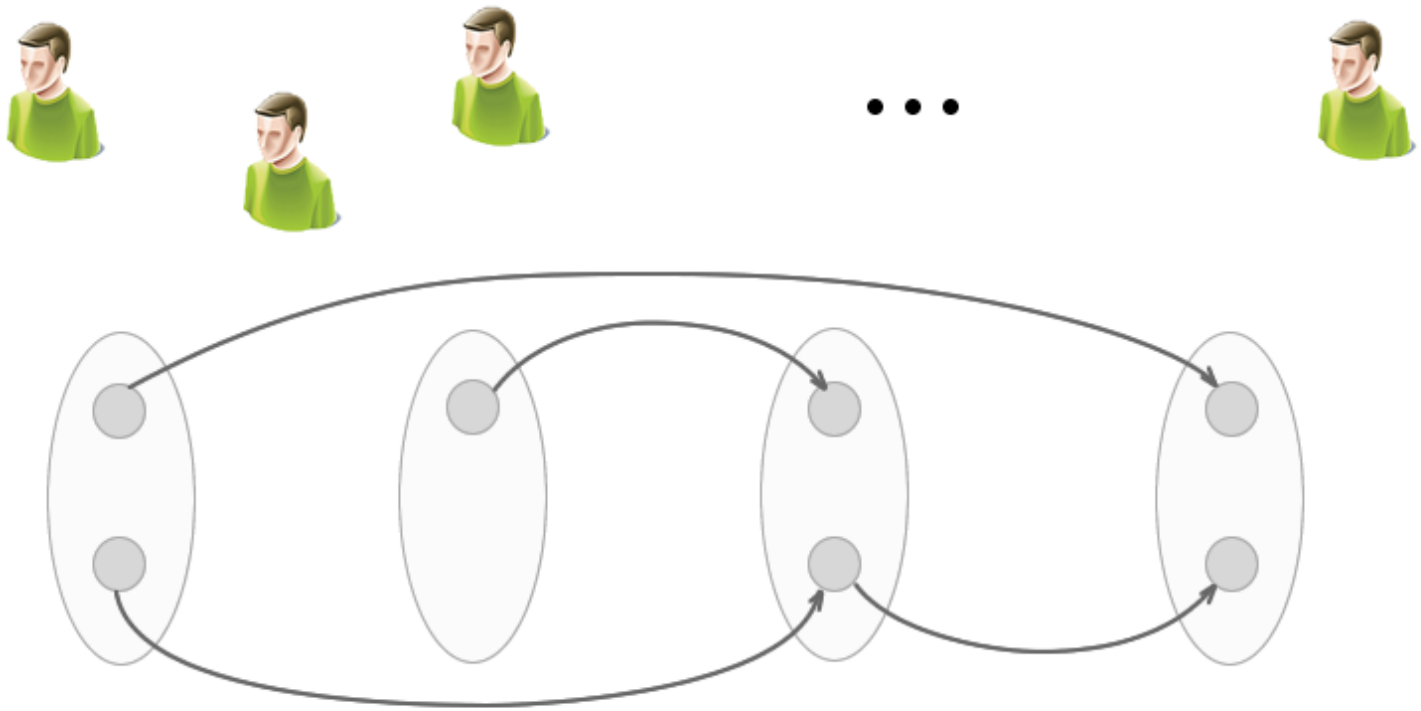
Temporal graphs and $L^*_{\mathcal{H}}$ in MAS

an *action* a_i consists of a **set** of **labelled** action nodes



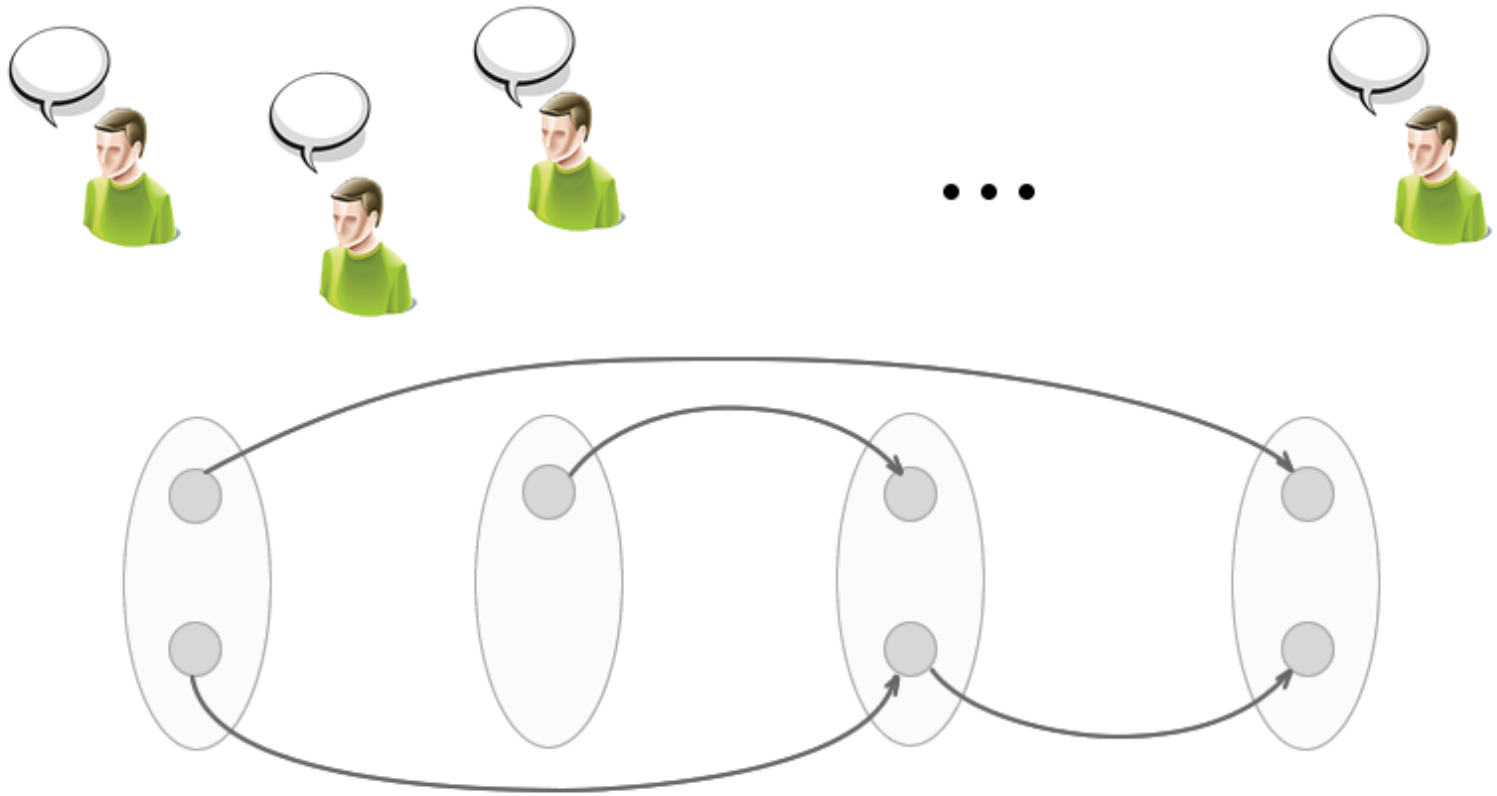
Temporal graphs and $L^*_{\mathcal{H}}$ in MAS

quality edges created/destroyed by action nodes



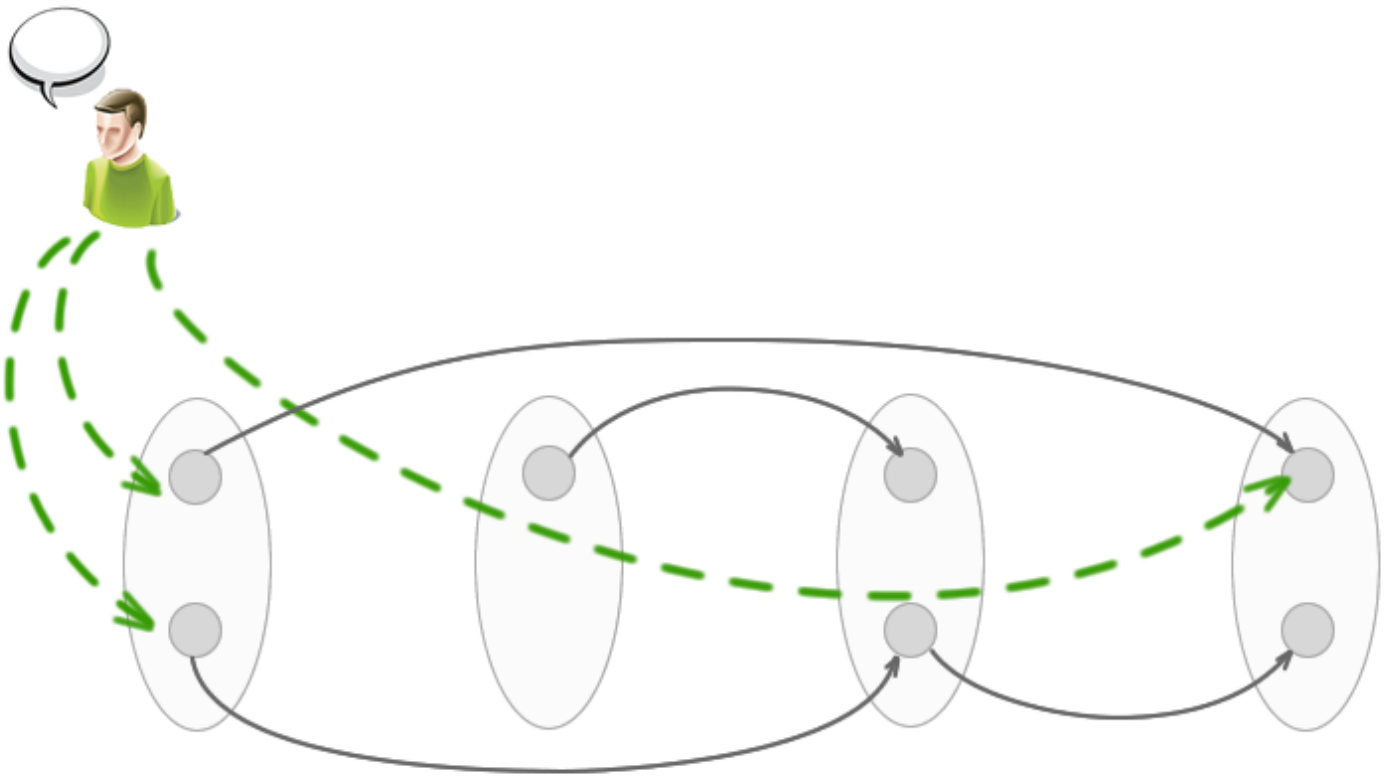
Temporal graphs and $L^*_{\mathcal{H}}$ in MAS

each player i has a **goal** φ_i. The goal is satisfied iff $\|\varphi_i\|^E_{\mathcal{H}} \neq \emptyset$



Temporal graphs and $L^*_{\mathcal{H}}$ in MAS

Deviation: Is a player incentivised to **change** his action, given the actions of others ?



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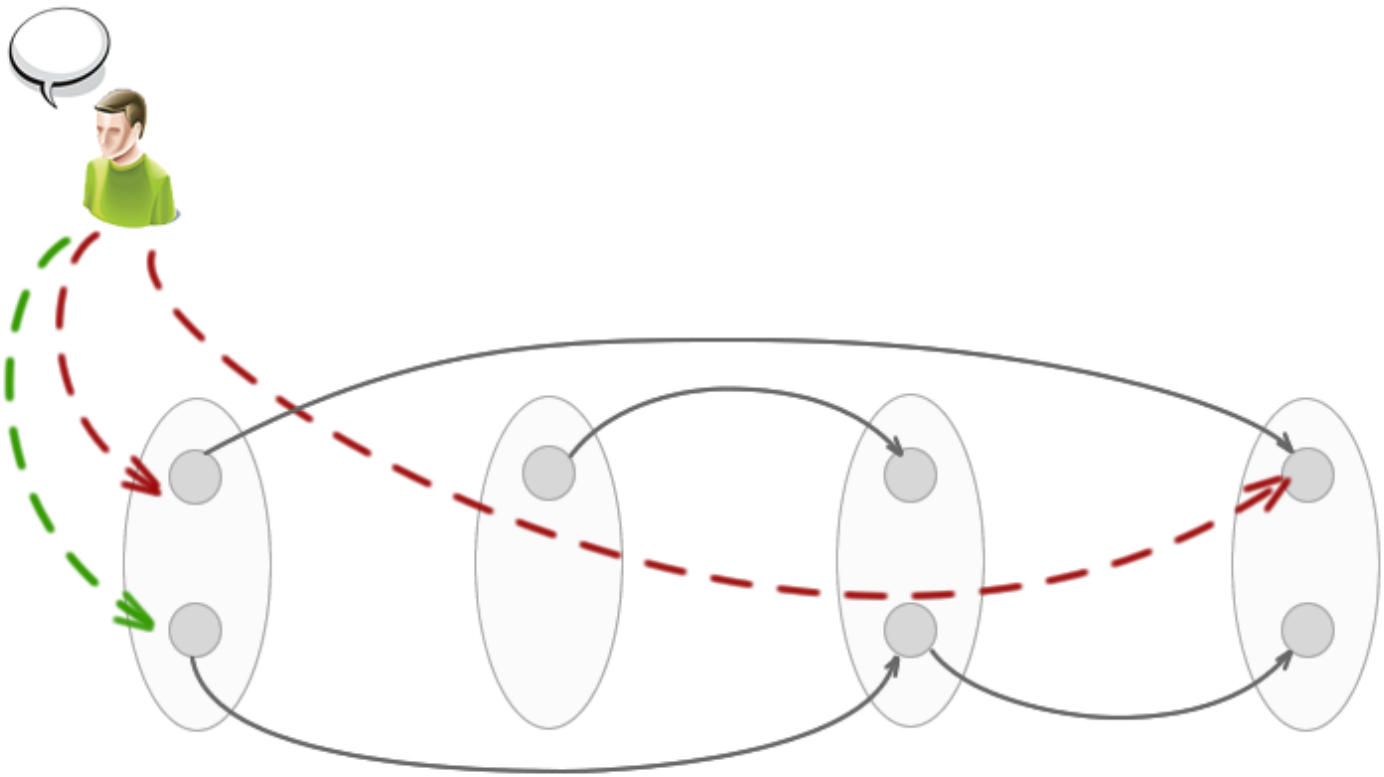

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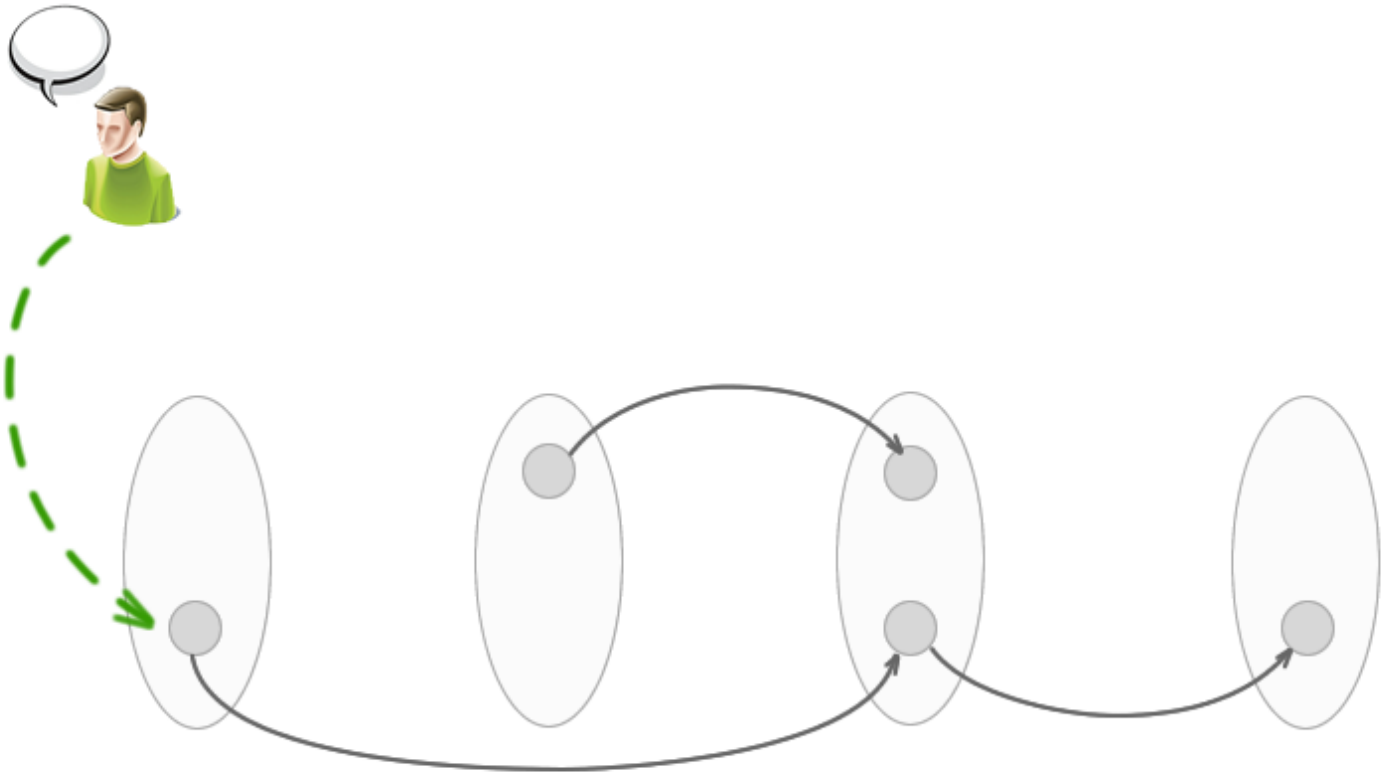

Temporal graphs and $L_{\mathcal{H}}$ in MAS

- *Remark:* The goal satisfaction of a player is affected by what **other players** do
- *Question:* What is the **proper** action each player should take, in order to **satisfy his goal** ?

Temporal graphs and $L_{\mathcal{H}}$ in MAS

An action profile $(a_1, \ldots, a_n)$ is a:

Temporal graphs and $L_{\mathcal{H}}$ in MAS

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- **Nash Equilibrium** (NE) iff there is no individual player that can **deviate**

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- **Strong Nash Equilibrium** (SNE) iff there is no **coalition** C of players that can **jointly deviate**

Complexity results (for $L_{\mathcal{H}}$ with propositional symbols):

- The **verification** problem: coNP for NE and SNE
- The **synthesis** problem: Σ_2 for NE and SNE

Applications of temporal graphs and $L_{\mathcal{H}}$

- Specifying **time-dependent** device behaviour in **intelligent buildings**
- Data mining logs from HPC systems
- Specifying **optimal** behaviour in **MAS** (Multi-Agent Systems)

Practical application: intelligent buildings

Specifying **time-dependent** device behaviour in intelligent buildings

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In it's initial phase, DSL was developed under the **FCINT POSCCE project** (Ontology-based Service Composition Framework for Syndicating Building Intelligence)

Conclusions

- **Temporal graphs** are suitable for storing information about the history of a domain
- **The language** $L^*_{\mathcal{H}}$ can be used to capture classes of system properties, that are involved in complex temporal relation.